3 Definition of the problem

Decentralized production systems are comprised by autonomous companies, in a value chain, that act autonomously and manage their own private information in order to prevent opportunistic actions of others. Inefficiency may settle in those systems as a consequence of lack of coordination among the companies participating in the value chain. The coordination of their actions may be lost due to the absence of the necessary private information flow and to the conflicts of interests between the companies. Coordinating*contracts* are proposed for overcoming managerial difficulties resulting from information flow obstruction and conflicting individual companies interests. In the supply chain, those contracts serve the purpose of approaching the system's optimal total performance. Ruling out managerial skills and other companies' individual characteristics, it is reasonable to assume that maximum system performance can be achieved under central planning where the companies are considered manufacturing units coordinated by a single entity, perhaps the owner or his proxy manager, who has full power over the units and access to all necessary information.

Customarily, manufacturing companies take a hierarchical planning approach to their production planning problem, distinguishing at least three time frames, namely: long-term, medium-term and short-term. Production planning is a varied and complex process not to be discussed in this dissertation. However, to frame this study, and to set forth some terms a brief, presentation of the simplified hierarchical planning process considered is in order. Long-term decisions are usually of a strategic nature and involve issues such as capital investments for balancing the supply chain members' capacities. They rely heavily on information external to the contracting companies, including competitors' moves. Medium-term decisions require coordination of capacity adjustments, such as available workers, small equipment additions, subcontracting, and supplies availability. These decisions concern production coordinating-contracts and require information (usually uncertain) on near-future demand. Shortterm decisions related to production planning are essentially when (in a fine scale) and how much to produce of what. They are based on firm orders, and hence on more certain information. Long-term production planning decisions are out of the scope of this study because they are usually heavily influenced by qualitative external information that is hard to model, and also because their direct linkages to medium- and short-term decisions are not clear.

The problem to be addressed in this study is the coordination by means of formal coordinating-contracts that can be used in a supply network of autonomous manufacturing companies that face uncertain market demands and have asymmetric information. The purpose of this coordination is to optimize the supply network's expected performance, which is assumed to be the sum of the companies' profits. Because the autonomous companies are free to reject the contract, it must improve the individual performance of each company through an acceptable partition of the gain that results directly from the contract itself. For reducing the inefficiencies that tend to arise in the supply network, a type of contract is designed with the purpose of coordinating the companies' medium-term capacity decisions. So, the problem in this study involves a hierarchical planning problem in each company because the medium-term capacity decisions must take into account their consequences on the company's short-term performance.

Here, market demands are considered to be continuous and probabilistically known. This exacerbates the complexity of modeling the hierarchical planning problem for each company. Thus, in order to obtain a tractable model, the study is restricted to a supply network comprised by a dyad of manufacturing companies which as described below together with their economic environment. Again for sake of tractability, a single period planning is considered, and no residual value is considered for left-over stock or unused capacity. Since the contract proposed intends the coordination of the companies' medium-term capacity decisions, in some manufacturing settings the problem to be solved is essentially the evaluation of the coordinating-contract efficiency and improvement. Efficiency evaluation will be made by comparing the dyad's performance under the contract with the dyad's one under central planning, while in the improvement evaluation the dyad's performances under and without the contract are compared. Clearly, the dyad's performance under the contract cannot be worse than the performance without contract, or better than the one under central planning.

A detailed description of the manufacturers in the dyad is presented in Section 3.1, which includes how they relate to each other with and without the contract, and, also, how each company interacts with its market. The possible inefficiencies of uncoordinated planning are discussed in Section 3.2, while Section 3.3 discusses some important issues on designing a coordinatingcontract. The proposed contract is detailed in Section 3.4, and Section 3.5 closes this chapter detailing the approach taken for performance evaluation.

3.1 Characterization of the supply network

The supply network considered is a dyad comprised by two autonomous manufacturing companies, whose final product can be either a consumer good, or an intermediate product in a larger value chain. Hereafter, the buyer company and supplier company will be called, respectively, Buyer and Supplier, while the supply network will be called Buyer-Supplier dyad, or simply dyad. Recall that, according to the convention adopted by Cachon (2003), the Buyer and Supplier are considered, respectively, to be male and female in this work.

Since both companies are manufacturers, they must address their medium-term decisions on production capacity to fulfill future orders from their potential customers. This is carried out taking into account the demand information known only probabilistically. Because the medium-term decision is assumed to be on overall capacity, only aggregate demand is relevant at this stage, even if the output is multi-product. The companies face a limited random demand that is partially lost if there is not sufficient medium-term production capacity, while unused capacity and production surplus have no residual value for the companies. The Buyer's aggregate final product and his aggregate relevant single input will be referred to as *product* and *material*, respectively. For the Supplier, aggregate output and input will be called, respectively, *material* and *raw material*. Hence a single intermediary aggregate product –the material– is the sole possible trade connection between the two manufacturers. The companies trade independently their input and output in their respective markets at market-given prices (see Figure 3.1).

The material and product markets are assumed to be in perfect competition, so the companies' participation in them is such that they do not influence the deterministically known market, and market prices are symmetric information for the companies and determined exogenously to them. The input markets for both companies have the ability of filling any order at market price. Consequently, each company can buy from the market as much of its input requirement as necessary to carry out the production that maximizes its performance. In the short-term, both companies operate in a make-to-order fashion, and hence their demands –assumed to be only probabilistically known in the medium-term– become deterministically known in the short-term. This is contrary to the usual microeconomics perfect-competition assumption of unlimited demand. However, in the business context, that is a quite plausible assumption if the company has a limited sales-force (see, for example, Lodish et al. 1988) and must spend effort for selling its products to the market at a deterministically known and fixed market-determined price.



Figure 3.1: Production system comprised by autonomous companies trading in the market.

Due to the autonomy of the companies, each one of them holds back private information in order to prevent opportunistic actions from the other. It is considered that Buyer has significant advantage on product market-demand information, due to be closer to the dyad's final customers than Supplier (see, for example, Özer and Wei, 2006), and he does not know the material demand-information from his competitors to Supplier. So, the material and product demand-information is assumed to be asymmetric between the companies. Asymmetrical information on the capacity and production costs is an important assumption, particularly, in situations in which there is no previous dealing that could provide information for the other party to estimate these costs. Knowing the other party's costs is a key issue to discount negotiation, and hence, in self-interest, none company will exchange this information voluntarily.

Each manufacturer adjusts his medium-term production capacity level by means of "soft" expansion (i.e. without investment in fixed assets), such as increases in manpower and outsourcing (Jin and Wu, 2007). The options for expanding the "soft capacity" are increasingly costly, so the capacity cost-function can be adequately represented by a strictly convex function. Production capacity is a medium-term decision and, thus, it is given in the short-term. Since unused capacity and left-over stock are assumed to have no residual value, the short-term marginal cost of capacity utilization is null. Hence, if unit production profit is positive, the company's short-term decision will be to produce, and sell, as much as possible (i.e. up to the minimum between the demand and capacity). Short-term marginal cost is assumed to be constant (i.e. constant unit variable-cost) for two main reasons. First, assuming that marginal cost to be variable would only complicate the determination of the short-term optimal decision (a deterministic problem) without adding much realism, or contributing to the study's purpose of evaluating the type of contract proposed. Second, the main portion of the unit variable-cost usually corresponds to such things as materials, energy and other inputs with quantityindependent unit prices.

To simplify the analyses, the rate of capacity consumption, as well as the rate of input consumption, per unit produced will be assumed to be unitary for both manufacturers. This does not change the essence of the problem because it is equivalent to a change of measuring units. Initial inventories pose no additional difficulties, since a single planning period is being considered. Any initial inventory can be deducted from the product demand or input necessity yielding a zero initial inventory problem.

3.2 Inefficiencies in the Buyer-Supplier dyad

The autonomous companies acting independently in the market, buying their inputs and selling their products at market prices, may become jointly inefficient principally because they have distinctively different financial incentives that may be in conflict. So, the phenomenon known as double marginalization (Spengler, 1950) may naturally occur in a decentralized decision situation. Double marginalization will lead Supplier and Buyer to set their capacities at different levels than it would be necessary for achieving all the profit each one could have if they acted in concert.

Indeed, assuming each company acts independently according to its own interest, Supplier and Buyer set their capacities so as to maximize their own expected profits given their own market prices (i.e. they will set their capacities to a value such that marginal expected cost equals marginal expected revenue). However, if they act in that fashion, it may happen that Supplier can still increase her expected profit by augmenting her capacity and selling more to Buyer by granting him a discount. Note that in spite of the market-determined fixed-price assumption of perfect competition, the marginal expected revenue is not constant because the Buyer's material order is an additional probabilistic demand to the Supplier, so price elasticity may be present. If, even at the discount price, her marginal expected revenue is still larger than her marginal expected cost (including the variable costs and the capacity cost), she will be increasing her expected profit. With the discount price, Buyer's marginal expected cost drops below his marginal expected revenue, this, in turn, allows him to increase his expected profit by increasing his capacity.

Therefore, the companies aiming at coordinating their capacity decisions can be beneficial to both parties. In central planning double marginalization can be avoided because Supplier and Buyer may be forced to increase their capacities until maximum joint expected profit is achieved. The coordinatingcontracts are proposed in a decentralized system to induce the coordination among the companies' actions such that their joint performance, as far as possible, approaches the central planning's performance. Contrary to central planning, where the planner entity may have all the necessary information for coordinating capacities, in independent planning no private information flows between the parties. Therefore, Supplier cannot set the right price for selling to Buyer, nor set her capacity for the right probabilistic demand Buyer will generate.

3.3 Designing a contract for the Buyer-Supplier dyad

The companies may be interested in establishing a contract for trading the material, with alternative sources to the market, if it allows them to overcome, at least, partially, their inefficient capacity decisions and receive part of the dyad's gain obtained by the contract. If the companies are autonomous, none is forced to accept the contract, so it will have to be worthy for both companies, that is, it must be advantageous for each party in the sense of each one being better off than acting independently in the market. Because no company controls the decisions in the dyad, the actual distribution of the gain derived from the contract coordination must be a consequence of a negotiation process. In this process, the information and the clout that each party possesses are key factors. Consequently, besides rendering an improvement for the companies' performances and for the dyad's gain, it is highly desirable that a coordinating-contract is capable of partitioning it arbitrarily.

The design of a contract should take into account certain aspects that companies would consider before undersigning it, such as the contract must be writable and executable at a cost not larger than the benefit that it can bring about. Also, the terms of the contract must address the exchange of information between the companies so as to eliminate or, at least, not stimulate opportunistic behavior of any party. Sharing private information may be required for attaining the coordination that the contract intends to induce. This information must be provided by one party to the other in a credible and voluntary manner. Therefore, it is imperative that the informing party does not lose by providing the information or can obtain some gain by misinforming the other party. In what follows the basic mechanism of the coordinating-contract to be proposed will be explained in an informal way looking from each side's point of view, while its formalization is left to the next section.

From Buyer's point of view, his expected profit could be increased if his short-term marginal production cost (which does not consider his capacity cost) were diminished. This would be possible if he could buy the material from Supplier at a price lower than the market price. If this happens, no matter what medium-term capacity decision he had set, Buyer would always be better off, and then he would be willing to consider Supplier as his primary source. However, if his medium-term capacity decision, made at market price, is limiting his expected sales, he could gain even more by acquiring additional capacity. Since his entire medium-term marginal expected cost curve drops, he will be willing to increase his medium-term-capacity up to the point where it equals the marginal expected revenue. Indeed, by increasing his capacity, Buyer will be able to deliver a larger amount of product to the market, and thus, to increase his expected profit. Summing up, Buyer may be able to gain more than the amount Supplier has transferred to him by granting the discount.

From the Supplier's point of view, if she has fixed her capacity to the point where her marginal expected revenue at market price is equal to her marginal capacity cost, then she will always lose the amount she transferred to Buyer via the discounted price. However, if the discounted price still leaves a margin over Supplier's expected variable production cost, then, even leaving fixed her capacity, her profit may increase due to the additional Buyer's (probabilistic) demand. Hence, at this point, the dyad may have obtained a net gain because Buyer may have gained more than the transferred amount, and Supplier may have gained for the increased Buyer's demand. However, if Supplier's marginal (relative to her capacity) expected revenue (net of expected variable production cost) is still larger than her marginal capacity cost, she may still add capacity and further her profit.

The above discussion leads to the possibility of improving the dyad's performance if Supplier offers Buyer a discount. This is a well known result for a manufacturer-retailer dyad in many different settings where double marginalization occurs (Serel et al., 2001; Cachon, 2004; Cachon and Lariviere, 2005; Jin and Wu, 2007; Erkoc and Wu, 2005; Cachon and Lariviere, 2001; and Özer and Wei, 2006). The contract to be designed intends to produce this performance improvement for a manufacturer-manufacturer dyad. This case differs from the case of a manufacturer-retailer dyad because once Buyer decides on his medium-term capacity, his future sales are limited to that

capacity. In contrast, in the case of a retailer, after learning on the actual demand he could still buy from the spot market for meeting the excess demand.

If the contract considers just the discount price Supplier offers to Buyer, he will be enticed to inflate as much as possible his order forecast so as to guarantee a virtually unlimited source of low priced material. If the contract states that Supplier will reward Buyer with a discount for a commitment to purchase no less than an agreed quantity, then Buyer can be dissuaded to inflate his order size forecast. However, this may turn the contract uninteresting to Buyer because the risk of idle capacity that was borne by Supplier is entirely transferred to Buyer. Furthermore, any unused part of the capacity Supplier has added for meeting Buyer's commitment may be used for selling to the spot market. So rather than imposing full loss of the units committed, but not ordered by Buyer, the contract can state some penalty for these units. That is in the spirit of popular contracts such as the DR (deductible reservation) contract, take-or-pay contract, or the buy-back contract.

The demand information available to each party can be represented by probability distributions. It is assumed that Buyer's knowledge on his product demand is a continuous distribution. Supplier's knowledge about her material demand can be represented by her forecast (a continuous probability distribution) for the market demand originated from all her potential customers, except Buyer, and the Buyer's forecast for his future order (i.e taking into account his already made capacity decision). In case Buyer has just a few competitors, this order information may be sensitive because it reveals Buyer's future intentions and thus may be of strategic importance for competitors. This aspect is very involved and peculiar to each case and will not be pursued here. In addition, Buyer's forecast for his future order size is unverifiable private information needed for coordination and, thus, must be truthfully passed to Supplier. Consequently, the contract must not lead him to misrepresent this information so as to manipulate the Supplier's decision in his own favor.

Though Buyer's additional demand is already beneficial for Supplier, she would gain even more if she could set her capacity closer to the actual need, i.e. if she can reduce her uncertainty on the Buyer's order size. As mentioned above, this will be accomplished if Buyer passes truthfully to her his order forecast. Though Buyer is not rewarded for passing his true forecasted order probability distribution, he has no incentive to distort that information. Indeed, by inflating his forecast he could cause damage to Supplier by inducing her to build excess capacity. However, he will gain nothing because the discount is not applicable to the units ordered beyond and above the contracted reserved capacity. For the same reason, he can injure Supplier by deflating his order forecast, thus leading her to build less than optimal capacity, but again he will not derive any benefit.

Therefore, the reward-penalty scheme described above presents rewards for both parties, and then it could really induce the sharing of capacity risk faced by both companies in the Buyer-Supplier dyad. In fact, Buyer would be sharing the Supplier's risk of negative payoff for reserving capacity to fulfill his purchase commitment and, consequently, the Supplier now would have an incentive to increase her medium-term production capacity. Since that none of the companies is a leader, or a follower, the contract designed is centered in the Buyer-Supplier dyad's performance rather than on the performance of any particular company. Hence, the commitment/reservation level, the discount over the units committed and the penalty for the units committed but not ordered will be the contract parameters. What parameter values should the contract contain for attracting the participation of both companies producing a win-win situation? This is the main question to be explored in the analysis of the contract. The remaining of this chapter provides further details of the contract, and sketches the analysis to be made.

3.4

A capacity reservation contract with reward-and-penalty

The capacity reservation contract proposed, and that considers reward and penalty, assumes that Supplier builds (in the medium-term), at least, the capacity-reservation commitment level (R) that will be purchased, partially or entirely, by Buyer in a future epoch (short-term). At this future date, Supplier will grant to Buyer a contract discount (d) on each unit he orders up to R, and Buyer will pay her a penalty (t) for each unit of R he fails to order. Thus, an instance of capacity reservation contract with reward-and-penalty is denoted by $\zeta = (R, d, t)$, where the per unit discount is given in relation to the spotmarket price and the per unit penalty charged to Buyer can be considered as a "reservation fee", if it were paid up front, that is not reimbursable by Supplier.

The terms of the agreement stipulated in the contract, as well as the specific values for the contract parameters, are agreed by both parties before they know their demand with certainty. The contract parameters can be set, for example, in a negotiation process between the companies in a previous phase, or simply as a take-it-or-leave-it proposal offered by one of the companies at the medium-term epoch. It is assumed that Supplier is somehow forced to comply with the terms of the contract (compliance may be verified through legal accountability because capacity is often observable). That is, Supplier will truly build, at least, the committed capacity level and will give priority to the Buyer's material order. Then she can use the remaining capacity for serving the other customers' orders at market price, if there is demand (see Figure 3.2).



Figure 3.2: A possible situation for Supplier's production capacity under the capacity reservation contract with reward-and-penalty.

Let $\zeta^{\circ} = (R^{\circ}, d^{\circ}, t^{\circ})$ be the capacity reservation contract with rewardand-penalty agreed previously by the companies. The following sequence of events details how the companies make their medium-term and short-term decisions under ζ° , which are sketched in the Figure (3.3).

FIRST STAGE (medium-term) – CAPACITY DECISION

- Buyer sets his medium-term production capacity based on the contract parameters $(R^{\circ}, d^{\circ}, t^{\circ})$, deterministic market prices, his forecast for the product demand (represented by a probability distribution), and his private knowledge on the production-capacity cost and variable costs. He informs Supplier about a forecast of his order quantity (represented by a probability distribution).

- Supplier sets her medium-term production capacity, at least, at the contract commitment level (R°) . Similarly to the Buyer, she decides on her capacity taking into account the contract parameters, the forecast for her material market-demand, her private information on costs, and also, the previously communicated Buyer's material-order forecast.

SECOND STAGE (short-term) – PRODUCTION QUANTITY DECISION Later on, at the short-term, Buyer and Supplier learn the actual values of their market demands realizations (x° and z° , respectively) and they decide on how much to produce so as to maximize their short-term profits (i.e. profits given their fixed capacities).

- Taking into account the observed demand, and the available capacity as decided in the medium-term, Buyer decides how much to produce (in this case, as much as he can produce and sell) and determines his material requirement. Then, he places his firm material order (y°) with the Supplier, and buys any possibly remaining material requirement from the spot market.

- Knowing Buyer's firm order (y°) and her actual market demand (z°) , Supplier decides how much to produce (analogously to Buyer, as much as she can produce and sell), delivers Buyer's firm order, and sells the remaining units to serve other customers in the spot market.

- Supplier gives the (fractional) discount d° over the material's market price p_m for each unit delivered to Buyer, and if Buyer did not order the entire committed quantity, she charges him the penalty t° for each unit committed that was not ordered. So, Buyer pays to Supplier the following amount: $(1 - d^{\circ}) p_m \min\{y^{\circ}, R^{\circ}\} + t^{\circ} (R^{\circ} - \min\{y^{\circ}, R^{\circ}\})^+$.

Note that the decision to undersign an instance of the contract is based on the forecasted demand information (probability distributions) and is made before each company decides how much to produce. So, the contract intends to coordinate medium-term capacities, and will be viable only if it leaves both companies better off when compared to the without-contract situation.

3.5 The approach for analyzing the contract proposed

The analysis of the capacity reservation contract with reward-penalty is carried over the optimal contracts. A contract is considered to be optimal if it maximizes the Buyer-Supplier dyad's expected profit, which is simply assumed as the sum of the companies' maximal expected profits under the terms of the contract. The assumption that management can always find the optimal decision under the available information is made, as usual, to simplify the analysis and to eliminate all performance factors except the one of interest, namely, the performance of the contract. As indicated above, to obtain the dyad's expected profit under a contract, the Buyer's and the Supplier's capacity problems are solved respectively in this sequence because Supplier requires Buyer's material order forecast distribution. So, to find the optimal contracts for the dyad, the companies' medium-term capacity problems are solved separately.

Of course, how the companies go about obtaining contract parameter values that lead to the dyad's expected improvement is of utmost practical



Figure 3.3: Medium- and short-term stages to make decisions under the capacity reservation contract with reward-and-penalty.

interest. If one company (usually called "leader") has full knowledge about the other company's costs and demand (usually called "follower"), it is possible to devise a game whose equilibrium solution is the contract parameter-tuple (see, for example, Cachon, 2003). One can also assume that several proposals can be offered by one party for the other to choose (Jin and Wu, 2007; Cachon and Lariviere, 2001; and Özer and Wei, 2006). However, the full-knowledge assumption is often unrealistic. In the asymmetrical information case, a more sophisticated means is to concoct a game where one party learns about the other's conditions and expectations exchanging proposals, counter-proposals, or revised proposals (see, for example, Dudek and Stadler, 2005).

The main purpose of the analysis here proposed is to appraise the theoretical potential of the proposed contract, under different environments, taking into account the randomness of the market demand that each company faces when making its medium-term capacity decision. The market demand, which is represented by a continuous random variable, is the unique stochastic information considered by the companies when deciding their medium-term capacity. Though, in practice, evaluations based on discrete scenarios are more common, the continuous representation provides a finer picture of the demand, and using a known distribution of few parameters, qualitative interpretations are easier than with a discrete representation. Even so, closed results are not easy to obtain and specific optimization interactive methods can be required for producing numerical solutions.

The issues to be investigated are the level of coordination that can be achieved in the Buyer-Supplier dyad, if any, and how the contract can share the gain between the companies. The contract negotiation process, or other mechanism, that the companies could carry out for agreeing on the contract will not be examined in this analysis. It is true that the main difficulty in contract design is to obtain a contract that simultaneously encompasses all desired features. Notwithstanding this, the analysis of a contract with realistic aspects is of great interest because, if it turns out to provide non-optimal, but yet reasonable gains, it may be implementable using the approximate information the parties have.

In the sequel, important assumptions made in the analysis of the contract proposed are specified. In addition, the problem of determining the maximal expected profit for a manufacturer is sketched, while its detailed formulation and solution will be presented in the next chapter. In addition, the benchmarks to be considered in the evaluation of the contract are presented, as well as the performance measures are defined.

3.5.1 Modeling assumptions

In the contract situation, Supplier receives information from Buyer about his order forecast. Since Buyer receives a discount just for the units he purchases up to the capacity commitment level (R), two different possibilities may be considered for the case his material requirement happens to exceed R, namely: (i) Buyer will inform Supplier only his forecast for his order up to R and buy all the excess requirement from the market at the known market price, and (ii) along with his order forecast, Buyer will also inform Supplier the forecasted material requirement in excess to the commitment level (R)(i.e. his entire material requirement) and will, as much as possible, buy it from Supplier, at market price. In contrast, the first possibility, per se, confers credibility to excess requirement information that Buyer provides because he cannot gain by manipulating this information.

The main difference between these possibilities is that, in the second one, Buyer's excess requirement is added to Supplier's demand improving the dyad's expected performance, and hence the attractiveness of the contract. For the same reason it also makes the comparison with the central planning performance easier to perform and to interpret. Indeed, under central planning it is implicit that Supplier has priority for supplying Buyer's material requirement. However, to consider Buyer buys the excess requirement from Supplier leads to a more complicate modeling because the additional quantity would be bought at market price. So, determining Buyer's expected profit must be distinguished if the units are bought at discount price, or at market price. Thus, to simplify the calculations to get that profit, in the models to be developed for the contract analysis (see the next chapter), will be considered Buyer informing to Supplier his entire material requirement but only buying from her up to the capacity commitment level given by the contract. Note that Supplier's expected profit could be sub-estimated if she has available capacity to serve Buyer's additional requirement. And, Buyer's manipulation of excess requirement information can improve his expected performance and how it affects the dyad's expected performance in an optimal contract is something yet to be investigated.

The main problem with the second possibility is that it assumes that Buyer provides Supplier his true beliefs on his excess requirement in a credible way. In other words, it is assumed that he does not distort this information to manipulate Supplier's capacity decision in self interest, and Supplier has a good reason to believe him. Thus, Buyer's manipulation of the excess material requirement distribution could lead Supplier to reject the optimal contract, for not giving her an acceptable improvement and, consequently, Buyer himself can be damaged if there is a contract acceptable by both parties under the true information. Therefore, the assumption of exchanging information in a credible manner is a reasonable one in the analysis of the capacity reservation contract here proposed.

In what follows it will be assumed that Buyer will truthfully inform Supplier about his forecast for his excess requirement. In addition, the Buyer's material order (including the requirement in excess of the committed purchase) and material market demand, are considered to be independent. This is not an unrealistic supposition because the markets served by the Buyer and the Supplier's other customers may be weakly related as, for example, if the material supplied is used by different industries, especially in the situation of a "buyer market" (excess capacity of the aggregate supply in the market) as assumed in this study.

On the other hand, Supplier may have an incentive for not building the capacity up to the commitment level (R) stipulated in the contract evaluated or, since the market material price is higher than the contract price, not using her capacity for filling the entire Buyer's material order. So, in the assessment of the contract, it is assumed that Supplier is forced to comply with the contract conditions. This means that she is somehow forced to build, at least, the capacity commitment level, use that capacity for fulfilling Buyer's order, and selling to market customers only the residual capacity. However, if the level of capacity can be verified, Supplier faces the possibility of being legally punished for infringing the contract because not filling Buyer's order up to the commitment level is obvious. So, she may be impelled to comply with the contract for avoiding foreseeable negative consequences, such as damage to her image that is important for future dealings.

3.5.2 Evaluating the manufacturer's performance

The manufacturer company's performance will be estimated through the expected profit, which is a function of the production-capacity decision. In deciding the medium-term production-capacity, each company considers the cost associated to it and the impact that it has in its operational profit. Since the market demand is only known probabilistically at the instant of making the capacity decision, that impact is evaluated by the expected operational profit that is determined taking the expectation over the market demand of the company's profit in the short-term. Getting the company's profit in the short-term, the medium-term capacity is considered fixed, but arbitrary, and the demand realization is considered be any value of the domain for the market-demand variable. Therefore, to evaluate the expected profit of each company involves a bi-level stochastic maximization problem in the production-capacity variable. Here that problem will be called *capacity problem*.

Let *B* and *S* represent, respectively, to Buyer and Supplier, while D_k , denotes the market demand for the company $k, k \in \{B, S\}$. For the situation in which the companies act independently in the spot market (i.e. withoutcontract situation), the company k's profit in the short-term is denoted by $\Pi_{k|C,d_k}(C)$, where *C* represents to the production-capacity variable (recall that it is considered fixed, but arbitrary, in the short-term) and d_k represents the demand realization (recall that it can be any value of the variable D_k), $k \in \{B, S\}$. The objective function for the company k's capacity problem corresponds to the expected profit $(EP_k(C))$ that can be expressed by the expected operational profit $(E_{D_k}[\Pi_{k|C,d_k}(C)])$ minus the medium-term production capacity cost $(\Psi_k(C)), k \in \{B, S\}$. The Figure 3.4 sketches the information involved in the capacity problem under the contract situation, in which each company must consider the given contract (ζ) in making its capacity decision. To distinguish between both situations, with- and withoutcontract, the subindex ζ will be add under the contract situation.



Figure 3.4: Information involved in the capacity problem under a given capacity reservation contract with reward-and-penalty.

3.5.3 Performance measures

Note that the relevant contracts are those that produce a gain for the dyad, ideally those that allow reaching the central planning performance, and also are viable for both companies, i.e. leave both better off in relation to the without-contract situation. So, the joint performance will be evaluated by comparing with two benchmarks representing extreme performances, which will be referred to as *independent planning* and *central planning*.

In independent planning, the companies make their capacity decisions independently, and trade solely with the market. Since any company can reject the contract, the performance under independent planning is a practical lower bound to the performance under the contract. In central planning, an entity that has access to the totality of the information held by the two companies and, also, full enforcement power decides on the capacity of each company aiming for maximizing the dyad's performance. Because there is no lack of information, then under the assumption that the central management can always determine the optimal action given the available information, the dyad's performance under central planning constitutes an upper bound for the dyad's performance under any contract.

Thus, the impact of the contract on the dyad is evaluated by comparing the dyad's expected profit under the contract, which is denoted by $EP_{D|\zeta}$, with the independent planning and central planning' performances that are denoted, respectively, by EP_{IP} , and EP_{CP}^* . Herein these relative measures are called, respectively, *improvement* $(\eta_{D|\zeta})$ and *efficiency* $(\xi_{D|\zeta})$, while the gain that the contract brings about for each company $(EP_{k|\zeta}, k \in \{B, S\})$, in comparison to its performance without contract $(EP_k, k \in \{B, S\})$, is called *surplus* and denoted by $\delta_{k|\zeta}, k \in \{B, S\}$. Of course for a contract to be viable, it must produce a positive surplus for every company. According to the given notation, those performance measures are defined by the following expressions: $\eta_{D|\zeta} = \left(\frac{EP_{D|\zeta}}{EP_{IP}} - 1\right) \cdot 100, \ \xi_{D|\zeta} = \frac{EP_{D|\zeta}}{EP_{CP}^*} \cdot 100, \ \text{and} \ \delta_{k|\zeta} = \left(\frac{EP_{k|\zeta}}{EP_{k}} - 1\right) \cdot 100, \ k \in \{B, S\}.$

The measures defined above allow to conclude about the issues being investigated in the analysis of the capacity reservation contract with rewardand-penalty, when they are applied over the viable and optimal contracts. In fact, the level of dyad coordination is given by the efficiency of any optimal contract, since all the optimal contracts lead to the same dyad's expected profit. Also, the distribution of the Buyer-Supplier dyad's gain is given by the possible values of $EP_{B|\zeta^*}$ and $EP_{S|\zeta^*}$, where ζ^* is a viable and optimal contract.