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A
Abreviações

BER	*Bit Error Rate*
BITE	Busca Iterativa
CDMA	*Code Division Multiple Access*
CP	*Cyclic Prefix*
DAB	*Digital Audio Broadcasting*
DFT	*Discrete Fourier Transform*
DMT	*Discrete Multitone*
DSL	*Digital Subscriber Lines*
DVB	*Digital Video Broadcasting*
ERB	Estação Rádio-Base
ESA	Estacionário no Sentido Amplo
FDM	*Frequency Division Multiplexing*
FFT	*Fast Fourier Transform*
IBI	*Inter-Block Interference*
ICI	*Inter-Carrier Interference*
IDFT	*Inverse Discrete Fourier Transform*
MIMO	*Multiple Input, Multiple Output*
ML	*Maximum Likelihood*
MMSE	*Minimum Mean Squared Error*
MSE	*Mean Squared Error*
OFDM	*Orthogonal Frequency Division Multiplexing*
PAPR	*Peak to Average Power Ratio*
RD	Ruído de Distorção
RI	Ruído de Interpolação
SC	*Single Carrier*
VA	Variável Aleatória
ZP	*Zero Padding*

B
Operadores Matemáticos

a	escalar
$\mathbf{a}$	vetor
$\mathbf{A}$	matriz
$\mathbf{a}[i]$	i-ésimo elemento de $\mathbf{a}$
$\mathbf{A}[i,j]$	(i,j)-ésimo elemento de $\mathbf{A}$
$\mathbf{I}_K$	matriz identidade de tamanho K
$\mathbf{0}_{K\times J}$	matriz de zeros com tamanho especificado pelo sub-escrito
$\mathbf{1}_{K\times J}$	matriz de 1's com tamanho especificado pelo sub-escrito
$\lvert\mathcal{A}\rvert$	cardinalidade do conjunto $\mathcal{A}$
$\lvert a\rvert$	módulo do escalar a
$\mathrm{diag}\{\mathbf{a}\}$	matriz com $\mathbf{a}$ na diagonal principal
$(\cdot)^*$	conjugado de um número
$(\cdot)^T$	transposta de uma matriz
$(\cdot)^\dagger$	pseudo-inversa de uma matriz
$(\cdot)^{-1}$	inversa de uma matriz
$(\cdot)^{\mathcal{H}}$	hermitiano de uma matriz
$\mathcal{E}[\cdot]$	valor esperado
$\mathrm{tr}\{\cdot\}$	traço de uma matriz
$\lVert\cdot\rVert_{\mathcal{F}}$	norma de Frobenius
$[\mathbf{A}]_{\mathrm{lin}\ i}$	i-ésima linha de $\mathbf{A}$
$[\mathbf{A}]_{\mathrm{col}\ i}$	i-ésima coluna de $\mathbf{A}$
$\lfloor a\rfloor$	arredondar para menor inteiro
$\lceil a\rceil$	arredondar para o maior inteiro
$\mathcal{N}(x,y)$	variável aleatória gaussiana de média x e variância y
$\Re\{\cdot\}$	parte real de um complexo

C
Cálculos para o MMSE

C.1
MSE

Seja o MSE temporal da estimação MMSE, i.e.

$$\mathrm{mse}_{\mathrm{mmse},n} = \mathcal{E}\left[\left\|\mathbf{h}_n - \hat{\mathbf{h}}_{\mathrm{mmse},n}\right\|^2\right]. \tag{C-1}$$

Usando (3-26), obtemos

$$\begin{aligned}
\mathrm{mse}_{\mathrm{mmse},n} &= \mathcal{E}\left[\left\|\mathbf{V}_n^{-1}\left(\mathbf{F}^{\mathcal{H}}\mathbf{S}_n^{\mathcal{H}}\mathbf{w}_{p,n} - \sigma\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}_n\right)\right\|^2\right] \\
&= \mathrm{tr}\Bigg\{\mathbf{V}_n^{-1}\Big[\mathcal{E}\left[\mathbf{F}_L^{\mathcal{H}}\mathbf{S}_n^{\mathcal{H}}\mathbf{w}_p\mathbf{w}_p^{\mathcal{H}}\mathbf{S}_n\mathbf{F}_L\right] \\
&+ \mathcal{E}\left[\sigma^2\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}\mathbf{h}^{\mathcal{H}}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\right]\Big]\mathbf{V}_n^{-1,\mathcal{H}}\Bigg\} \\
&= \mathrm{tr}\Bigg\{\mathbf{V}_n^{-1}\Big[\sigma\mathbf{F}_L^{\mathcal{H}}\mathbf{D}_n\mathbf{F}_L + \sigma^2\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\Big]\mathbf{V}_n^{-1,\mathcal{H}}\Bigg\}.
\end{aligned} \tag{C-2}$$

Aqui usamos $\mathcal{E}\left[\mathbf{w}_p\mathbf{w}_p^{\mathcal{H}}\right] = \sigma\mathbf{I}_{K_p}$ e $\mathcal{E}\left[\mathbf{h}_n\mathbf{h}_n^{\mathcal{H}}\right] = \mathrm{diag}\left\{\boldsymbol{\gamma}\right\}$, como já definido no texto. Seguimos identificando que $\mathbf{V}^{-1} = \mathbf{V}^{-1,\mathcal{H}}$ e reduzindo (C-2) a

$$\begin{aligned}
\mathrm{mse}_{\mathrm{mmse},n} &= \mathrm{tr}\Big\{\mathbf{V}_n^{-1}\sigma\mathbf{V}\mathbf{V}_n^{-1,\mathcal{H}}\Big\} \\
&= \sigma\mathrm{tr}\big\{\mathbf{V}_n^{-1}\big\},
\end{aligned} \tag{C-3}$$

onde usamos a definição de $\mathbf{V}$.

Considere agora o MSE da estimação de um determinado tom da resposta de freqüência do canal:

$$\mathrm{MSE}_{\mathrm{mmse},n}^k = \mathcal{E}\left[\left\|q_n^k - \hat{q}_{\mathrm{mmse},n}^k\right\|^2\right]. \tag{C-4}$$

Considerando (3-27), obtemos

$$\begin{aligned}\mathrm{MSE}^k_{\mathrm{mmse},n} &= \mathcal{E}\left[\left\|\mathbf{f}^k\mathbf{V}_n^{-1}\left(\mathbf{F}_L^{\mathcal{H}}\mathbf{S}_n^{\mathcal{H}}\mathbf{w}_{p,n} - \sigma\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}_n\right)\right\|^2\right] \\ &= \mathrm{tr}\Big\{\mathbf{f}^k\mathbf{V}_n^{-1}\Big[\mathcal{E}\left[\mathbf{F}_L^{\mathcal{H}}\mathbf{S}_n^{\mathcal{H}}\mathbf{w}_p\mathbf{w}_p^{\mathcal{H}}\mathbf{S}_n\mathbf{F}_L\right] \\ &+ \mathcal{E}\left[\sigma^2\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}\mathbf{h}^{\mathcal{H}}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\right]\Big](\mathbf{f}^k)^{\mathcal{H}}\mathbf{V}_n^{-1,\mathcal{H}}\Big\}. \end{aligned} \tag{C-5}$$

A solução de (C-5) é muito parecida com a de (C-2):

$$\mathrm{MSE}^k_{\mathrm{mmse},n} = \mathbf{f}^k\mathbf{V}_n^{-1}(\mathbf{f}^k)^{\mathcal{H}}. \tag{C-6}$$

C.2
Filtro de Wiener

Precisamos calcular a matriz $\boldsymbol{\mathcal{R}}_{\mathbb{YY}}$ e o vetor $\boldsymbol{\mathcal{R}}_{\mathbf{h}_n\mathbb{Y}}$ levando em conta as características do MMSE. Primeiramente, temos

$$\begin{aligned}\boldsymbol{\mathcal{R}}_{\mathbb{YY}} &= \mathcal{E}\left[\mathbb{YY}\right] = \mathcal{E}\left[(\mathbb{H}+\mathbb{W})^{\mathcal{H}}(\mathbb{H}+\mathbb{W})\right] \\ &= \underbrace{\mathcal{E}\left[\mathbb{H}^{\mathcal{H}}\mathbb{H}\right]}_{\mathcal{A}} + \underbrace{2\Re\left\{\mathcal{E}\left[\mathbb{H}^{\mathcal{H}}\mathbb{W}\right]\right\}}_{\mathcal{B}} + \underbrace{\mathcal{E}\left[\mathbb{W}^{\mathcal{H}}\mathbb{W}\right]}_{\mathcal{C}}, \end{aligned} \tag{C-7}$$

onde $\mathbb{H}$ é dado por (3-35) e $\mathbb{W}$ é dado por (3-49) e $\Re\{\cdot\}$ denota a parte real de um elemento. A solução de $\mathcal{A}$ é fácil pois já foi visto que $\mathcal{E}\left[\mathbb{H}^{\mathcal{H}}\mathbb{H}\right] = \boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}$. Partimos agora para a solução de $\mathcal{B}$ e $\mathcal{C}$.

$$\begin{aligned}\mathcal{B} &= -2\Re\left\{\mathcal{E}\left[\mathbb{H}^{\mathcal{H}}\mathbb{W}\right]\right\} \\ &= -2\sigma\begin{bmatrix} \Re\{\mathrm{tr}\{\mathbf{V}_n^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathcal{E}\left[\mathbf{h}_n\mathbf{h}_n^{\mathcal{H}}\right]\}\} & \cdots \\ \Re\{\mathrm{tr}\{\mathbf{V}_{n-1}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathcal{E}\left[\mathbf{h}_{n-1}\mathbf{h}_n^{\mathcal{H}}\right]\}\} & \cdots \\ \vdots & \ddots \end{bmatrix}. \end{aligned} \tag{C-8}$$

Sendo $\mathcal{E}\left[\mathbf{h}_n\mathbf{h}_{n-i}^{\mathcal{H}}\right] = \mathrm{diag}\left\{\boldsymbol{\gamma}\right\} \times \mathcal{R}_{h,\mathrm{t}}[i]$, simplificamos (C-8) para

$$\mathcal{B} = -2\sigma\mathrm{tr}\{\mathbf{V}^{-1}\}\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}. \tag{C-9}$$

Aqui consideramos os pilotos constantes no tempo, ou seja, $\mathbf{p}_i = \mathbf{p}_j\ \forall i,j$, o que implica em $\mathbf{V}_i^{-1} = \mathbf{V}_j^{-1}\ \forall i,j$.

Agora falta $\mathcal{C}$.

$$\mathcal{C} = \mathcal{E}\left[\mathbb{W}^{\mathcal{H}}\mathbb{W}\right] \tag{C-10}$$

Essa variável terá duas parcelas, representadas respectivamente por $\mathcal{C}_1$ e $\mathcal{C}_2$.

Consideramos de novo pilotos constantes no tempo. Temos

$$\begin{aligned}\mathcal{C}_1 &= \begin{bmatrix} \mathrm{tr}\left\{\mathbf{V}^{-1}\mathbf{F}_L^{\mathcal{H}}\mathbf{S}^{\mathcal{H}}\mathcal{E}\left[\mathbf{w}_{p,n}\mathbf{w}_{p,n}^{\mathcal{H}}\right]\mathbf{S}\mathbf{F}_L\mathbf{V}^{-1}\right\} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathrm{tr}\left\{(\cdot)_{n-M+1}\right\} \end{bmatrix} \\ &= \sigma\mathrm{tr}\left\{\mathbf{V}^{-1}\mathbf{U}\mathbf{V}^{-1}\right\}\mathbf{I}_M, \end{aligned} \tag{C-11}$$

e

$$\begin{aligned}\mathcal{C}_2 &= \begin{bmatrix} \sigma^2\mathrm{tr}\left\{\mathbf{V}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}^{-1}\right\}\mathcal{E}\left[\mathbf{h}_n\mathbf{h}_n^{\mathcal{H}}\right]\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{V}^{-1}\right\} & \cdots \\ \sigma^2\mathrm{tr}\left\{\mathbf{V}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}^{-1}\right\}\mathcal{E}\left[\mathbf{h}_{n-1}\mathbf{h}_n^{\mathcal{H}}\right]\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{V}^{-1}\right\} & \cdots \\ \vdots & \ddots \end{bmatrix} \\ &= \sigma^2\mathrm{tr}\left\{\mathbf{V}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{V}^{-1}\right\}\times\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}. \end{aligned} \tag{C-12}$$

Finalmente, combinando $\mathcal{A}$ com (C-9), (C-11) e (C-12), chegamos a

$$\begin{aligned}\boldsymbol{\mathcal{R}}_{\mathbb{YY}} = \boldsymbol{\mathcal{R}}_{hh,\mathrm{t}} \times \Big(1 - 2\sigma\mathrm{tr}\left\{\mathbf{V}^{-1}\right\} + \sigma^2\mathrm{tr}\left\{\mathbf{V}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{V}^{-1}\right\}\Big) \\ + \sigma\mathrm{tr}\left\{\mathbf{V}^{-1}\mathbf{U}\mathbf{V}^{-1}\right\}\mathbf{I}_M. \end{aligned} \tag{C-13}$$

A solução de $\boldsymbol{\mathcal{R}}_{\mathbf{h}_n\mathbb{Y}}$ é mais fácil.

$$\boldsymbol{\mathcal{R}}_{\mathbf{h}_n\mathbb{Y}} = \left[\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\right]_{\mathrm{col}\ 1} \times \left(1 - \sigma\mathrm{tr}\left\{\mathbf{V}^{-1}\right\}\right). \tag{C-14}$$

C.3 MSE para MMSE mais filtro de Wiener

Seja

$$\mathrm{mse}_{\mathrm{mmse+w},n} = \mathcal{E}\left[\left\|\mathbf{h}_n - \hat{\mathbf{h}}_n\right\|^2\right], \tag{C-15}$$

com $\hat{\mathbf{h}}_n = \mathbb{Y}\boldsymbol{\lambda}$ e com o filtro $\boldsymbol{\lambda}$ calculado especificamente para o MMSE. Usando (3-27), obtemos

$$\begin{aligned}\mathrm{mse}_{\mathrm{mmse+w},n} = \mathcal{E}\Bigg[\Bigg\|\mathbf{h}_n - \sum_{i=0}^{M-1}\lambda[i]\mathbf{h}_{n-i} + \\ - \sum_{i=0}^{M-1}\lambda[i]\sigma\mathbf{V}_{n-i}^{-1}\left(\mathbf{F}_L^{\mathcal{H}}\mathbf{S}_{n-i}^{\mathcal{H}}\mathbf{w}_{p,n} - \mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}_{n-i}\right)\Bigg\|^2\Bigg]. \end{aligned} \tag{C-16}$$

É fácil ver que dois dos termos do MSE resultante são dados por $\boldsymbol{\lambda}'\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\boldsymbol{\lambda}'$ e por $\sigma\sum_{i=0}^{M-1}\lambda[i]^2\mathrm{tr}\left\{\mathbf{V}_{n-i}^{-1}\right\}$. Há, porém, mais um termo, que chamaremos de

$\mathcal{D}$, dado por

$$\mathcal{D} = \mathcal{E}\left[\left\|\mathbf{h}_n - \sum_{i=0}^{M-1} \lambda[i]\mathbf{h}_{n-i} - \sum_{i=0}^{M-1} \lambda[i]\mathbf{V}_{n-i}^{-1}\sigma\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}_{n-i}\right\|^2\right]. \quad \text{(C-17)}$$

Notamos que podemos escrever

$$\mathbf{h}_n - \sum_{i=0}^{M-1} \lambda[i]\mathbf{h}_{n-i} = \sum_{i=0}^{M-1} \lambda'[i]\mathbf{h}_{n-i} \quad \text{(C-18)}$$

e

$$\begin{aligned}
\mathcal{D} &= -\sigma\mathcal{E}\left[\sum_{i=0}^{M-1} \lambda'[i]\mathbf{h}_{n-i}^{\mathcal{H}} \sum_{j=0}^{M-1} \lambda[j]\mathbf{V}_{n-j}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathbf{h}_{n-j}\right] \\
&= -\sigma\mathrm{tr}\left\{\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} \lambda'[i]\lambda[j]\mathbf{V}_{n-j}^{-1}\mathrm{diag}\left\{\boldsymbol{\gamma}\right\}^{-1}\mathcal{E}\left[\mathbf{h}_{n-j}\mathbf{h}_{n-i}^{\mathcal{H}}\right]\right\} \\
&= -\sigma\mathrm{tr}\left\{\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} \lambda'[i]\lambda[j]\mathbf{V}_{n-j}^{-1}\mathcal{R}_{h,\mathrm{t}}\left[\left|i-j\right|\right]\right\}. \quad \text{(C-19)}
\end{aligned}$$

Se considerarmos pilotos constantes no tempo, podemos retirar o termo $\mathbf{V}^{-1}$ do somatório. Obtemos

$$\begin{aligned}
\mathcal{D} &= -\sigma\mathrm{tr}\left\{\mathbf{V}_n^{-1}\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} \lambda'[i]\lambda[j]\mathcal{R}_{h,\mathrm{t}}\left[\left|i-j\right|\right]\right\} \\
&= -\sigma(\boldsymbol{\lambda}')^T\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\boldsymbol{\lambda}\mathrm{tr}\left\{\mathbf{V}_n^{-1}\right\} \quad \text{(C-20)}
\end{aligned}$$

Finalmente, conseguimos

$$\mathrm{mse}_{\mathrm{mmse+w},n} = (\boldsymbol{\lambda}')^T\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\boldsymbol{\lambda}' + \sigma\sum_{i=0}^{M-1} \lambda[i]^2\mathrm{tr}\left\{\mathbf{V}_{n-i}^{-1}\right\} - \sigma(\boldsymbol{\lambda}')^T\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\boldsymbol{\lambda}\mathrm{tr}\left\{\mathbf{V}_n^{-1}\right\}. \quad \text{(C-21)}$$

Na freqüência, tom k, obtemos

$$\begin{aligned}
\mathrm{MSE}_{\mathrm{mmse+w},n}^k = (\boldsymbol{\lambda}')^T\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\boldsymbol{\lambda}' + \sigma\sum_{i=0}^{M-1} \lambda[i]^2\mathbf{f}^k\mathbf{V}_{n-i}^{-1}(\mathbf{f}^k)^{\mathcal{H}} + \\
- \sigma\mathbf{f}^k\mathbf{V}_n^{-1}(\mathbf{f}^k)^{\mathcal{H}}(\boldsymbol{\lambda}')^T\boldsymbol{\mathcal{R}}_{hh,\mathrm{t}}\boldsymbol{\lambda}. \quad \text{(C-22)}
\end{aligned}$$