5
Conclusion

We have investigated the 2-categorical view of Proof Theory, a work already provided by Seely (17) and expanded in this dissertation. The first thing we notice when expanding Categorical Logic to 2-Category Theory, is the advantage of being able to represent reductions as 2-cells, what give us a categorical view closer to the proof-theoretical semantics. With this view we have concluded that

- conjunction can be seen as lax 2-adjoint to the diagonal 2-functor;
- disjunction can be seen as rax 2-adjoint to the diagonal 2-functor;
- implication cannot be seen neither as lax 2-adjoint nor as rax 2-adjoint to the diagonal 2-functor;
- $\perp$ cannot be seen as (lax) 2-initial object;
- Ekman's reduction cannot be 2-categorically represented.

If, in the 2-category $\mathcal{PT}$, we had represented every rex between two derivations by the same 2-cell, then the unicity of γ in the definition of 2-product (see p. 39) would have been trivially satisfied and then conjunction would be seen as lax 2-product. Analogously, disjunction would be seen as lax 2-co-product and implication as lax 2-exponential. This is what happens if we take rex as a preorder relation between derivations.

In contrast to the 1-categorical view of intuitionistic natural deduction, which collapses equivalent derivations into only one arrow, we hope that our ability to distinguish derivations and to categorically deal with reductions between derivations might help to understand a bit more the identity problem on normal derivations. At least this work contributed to a closer correspondence between the models of Category Theory and Proof Theory.

Two subjects for future research are the investigation of the categorical view of the Classical Natural Deduction and to expand the 2-categorical view of Proof Theory to cope with derivations with more than one premiss.