CETUC

Publicações em Antenas

Number 5 | October 2012

A Time-Domain Uniform Asymptotic Theory Applied to the Analysis of Pulse-Excited PEC Wedges

Cássio G. Rego

Flavio J. V. Hasselmann

GRUPO DE ANTENAS

PUC RIO

Publicações em Antenas

Number 5 | October 2012

A Time-Domain Uniform Asymptotic Theory Applied to the Analysis of Pulse-Excited PEC Wedges

Cássio G. Rego

Flavio J. V. Hasselmann

CRÉDITOS

Publisher:
MAXWELL / LAMBDA-DEE
Sistema Maxwell / Laboratório de Automação de Museus, Bibliotecas Digitais e Arquivos
http://www.maxwell.lambda.ele.puc-rio.br/

Editor:
Jose Ricardo Bergmann

Capa:
Ana Cristina Costa Ribeiro

A Time-Domain Uniform Asymptotic Theory Applied to the Analysis of Pulse-Excited PEC Wedges

Cássio G. Rego and Flavio J. V. Hasselmann, *Member, IEEE*

***Abstract*—In this letter, a time-domain version of the uniform asymptotic theory of diffraction, henceforth named TD-UAT, is introduced and is employed to obtain the fields scattered by a perfectly electrically conducting wedge illuminated by an arbitrarily oriented electric dipole. The results of TD-UAT are compared to those obtained from an implementation of the time-domain uniform theory of diffraction (TD-UTD), and its applicability is demonstrated.**

***Index Terms*—Time-domain uniform asymptotic theory (TD-UAT), transient electromagnetics.**

I. Introduction

The PROBLEM of the determination of the electromagnetic field due to a source radiating in the presence of a structure occurs in many practical situations, such as in the design of reflector antennas, in the prediction of the coverage of cellular systems, and in the design of radio links. In the first practical applications of the scattering problem, the sources operated in the steady-state modes. This fact justified the development of techniques, analytical and numerical, formulated in the frequency domain so that only one frequency could be analyzed at a time. In recent years, the need of high transmission rates in digital communication systems, the development of the short-pulse radar, and the interest in the effects of short electromagnetic pulses on communication and radar systems have promoted the research on the transient electromagnetic analysis. Many techniques of determination of the scattered fields radiated by pulsed sources have been developed since. To be more practical, the techniques for transient electromagnetic analysis must provide solutions directly into the time domain. There exist various methods for analytical analysis of transient electromagnetic problems. The spectral theory of transients (STT) has been developed to find the exact time domain solution for canonical geometries [1]–[4]. The exact solution to scattering by a straight wedge considering different types of illumination was obtained by Felsen [5]–[7]. The implicit time dependence of diffraction coefficients listed in these solutions (e.g., [7, eq. (3)–(5)] for the wedge problem), however, tends to render the evaluation of convolution integrals inherent to the calculation of the response to finite-energy pulse excitations very laborious. Nevertheless, this may be alleviated by performing early-time approximations to yield a simpler analytic alternative, as readily achieved via time-domain versions of asymptotic high-frequency domain methods of analysis of the scattered field by perfectly electrical conducting (PEC) bodies, which have been recently developed and applied to obtain the transient response of reflector antennas. These include the time-domain uniform geometrical theory of diffraction (TD-UTD) introduced by Pathak *et al.* [8]–[10] and the time-domain physical optics (TD-PO) introduced by Rusch and Sun [11]. These techniques sustain the simplicity of the physical optics currents and the simple ray picture of their frequency-domain counterparts. The TD-PO formulation, together with the development of time-domain-equivalent fringe wave currents (TD-FWCs) to correct the imperfections of the physical-optics induced currents near surface edges, has been implemented in a practical application within the framework of reflector antennas illuminated by pulsed sources [12], [13].

This letter deals with the canonical problem of the pulsed field diffraction by a perfectly electrically conducting wedge. A time-domain version of the uniform asymptotic theory of diffraction [14], henceforth named TD-UAT, is introduced herein, and it is used to obtain the fields scattered by the wedge when it is illuminated by an arbitrarily oriented electric pulsed dipole, which has not been previously reported in the literature. By an appropriate Fourier inversion of its formulation in the frequency domain, the TD-UAT wedge response, generating a finite energy scattered pulse, is obtained in a manner similar to its TD-UTD counterpart. Comparison of results obtained by both methodologies, for the case of field illuminations with rapid spatial variation near the edge, also reveals that higher-order diffraction effects are contained within the TD-UAT formulation, while the TD-UTD field description must be enhanced at times by the inclusion of slope-diffraction terms. The applicability of the TD-UAT introduced herein is also ascertained by verifying its reduction, away from shadow boundaries, to the time-domain version of the classical GTD derived elsewhere [15].

II. Formulation of the TD-UAT

The TD-UAT analytic response of a pulse-excited perfectly electrically conducting wedge is obtained by an one-sided

Manuscript received July 14, 2011; revised August 29, 2011; accepted September 01, 2011. Date of publication September 08, 2011; date of current version September 29, 2011. This work was supported in part by CNPq under Covenant 573939/2008-0 (INCT-CSF) and Project 300938/2008-0 and Fapemig under Projeto do Pesquisador Mineiro.

C. G. Rego is with the Department of Electronic Engineering, Federal University of Minas Gerais, Belo Horizonte 31270-901, Brazil (e-mail: cassio@cpdee.ufmg.br).

F. J. V. Hasselmann is with the Center of Studies in Telecommunications Studies, Catholic University of Rio de Janeiro, Rio de Janeiro 22451-900, Brazil (e-mail: flavio@cetuc.puc-rio.br).

Color versions of one or more of the figures in this letter are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LAWP.2011.2167590

inverse Fourier transform of its corresponding frequency-domain formulation and can be written in the form [8], [10]

$$\overset{+}{\mathbf{E}}^T(\mathbf{r},t) = \frac{1}{\pi}\int_0^\infty \mathbf{E}^T(\mathbf{r},\omega)e^{j\omega t}d\omega, \qquad \Im m(t) > 0 \tag{1}$$

where $\mathbf{E}^T(\mathbf{r},\omega)$ is the frequency-domain UAT field

$$\mathbf{E}^T(\mathbf{r},\omega) = \mathbf{E}^G(\mathbf{r},\omega) + \mathbf{E}^d(\mathbf{r},\omega) \tag{2}$$

with the modified geometrical optics and diffracted fields being respectively expressed by [14]

$$\mathbf{E}^G(\mathbf{r},\omega) = F^i(\omega)\mathbf{E}^i(\mathbf{r},\omega) + F^r(\omega)\mathbf{E}^r(\mathbf{r},\omega) \tag{3a}$$

$$\mathbf{E}^d(\mathbf{r},\omega) = -\left[D_s(\omega)E^i_{\beta_0'}(Q_E,\omega)\hat{\beta}+D_h(\omega)E^i_{\phi_0'}(Q_E,\omega)\hat{\phi}\right] \times A_d(s^d)e^{-jk_0 s^d}. \tag{3b}$$

In (3a), we have the incident field $\mathbf{E}^i(\mathbf{r},\omega)$ and the reflected field $\mathbf{E}^r(\mathbf{r},\omega)$, which is present in all observation regions as the wedge faces are extended beyond the reflection shadow boundaries. Transition functions $F^i(\omega)$ and $F^r(\omega)$ can be written as [14]

$$F^{i,r}(\omega) = \frac{1}{2}\left\{\mathrm{erfc}(C^{i,r}\sqrt{j\omega}) - \frac{e^{-j\left[\omega(C^{i,r})^2+\frac{\pi}{4}\right]}}{C^{i,r}\sqrt{\pi\omega}}\right\} \tag{4}$$

where $\mathrm{erfc}(\cdot)$ is the complementary error function [8] and

$$C^i = \epsilon^i\sqrt{\frac{1}{c}\left|s^d + s^i(Q_E) - s^i(P)\right|} \tag{5}$$

$$C^r = \epsilon^r\sqrt{\frac{1}{c}\left|s^d + s^i(Q_E) - s^i(Q_R) - s^r\right|} \tag{6}$$

are related to the so-called detour parameters of the frequency-domain UAT, $\xi^{i,r} = C^{i,r}\sqrt{\omega}$, with their respective distances s^d, $s^i(Q_E)$, $s^i(P)$, $s^i(Q_R)$, and s^r, as well as the shadow indicators $\epsilon^{i,r}$ [14]. Equation (3b) shows the diffracted field expressed in terms of the incident field components at the diffraction point, $E^i_{\beta_0'}(Q_E,\omega)$ and $E^i_{\phi_0'}(Q_E,\omega)$, the spreading factor

$$A_d(s^d) = \sqrt{\frac{1}{s^d(1+s^d/R)}} \tag{7}$$

and diffraction coefficients, which are given by

$$D_{s,h}(\omega) = -\frac{e^{-j\frac{\pi}{4}}}{2n\sqrt{2\pi}\sin\beta_0}\sqrt{\frac{c}{\omega}}\sum_{m=1}^{4}K_m^{s,h} \tag{8}$$

where R is the principal radius of curvature of the diffracted wavefront at Q_E [14], c is the velocity of light, n is defined by the wedge exterior angle $n\pi$, and

$$K_1^{s,h} = \cot\left(\frac{\pi+\beta^-}{2n}\right) \tag{9a}$$

$$K_2^{s,h} = \cot\left(\frac{\pi-\beta^-}{2n}\right) \tag{9b}$$

$$K_3^{s,h} = \mp\cot\left(\frac{\pi+\beta^+}{2n}\right) \tag{9c}$$

$$K_4^{s,h} = \mp\cot\left(\frac{\pi-\beta^+}{2n}\right) \tag{9d}$$

with

$$\beta^\pm = \phi \pm \phi_0. \tag{10}$$

All relevant parameters are indicated in Fig. 1. From (9), we can observe the well-known singularities of the diffraction coefficients on the shadow boundaries. These singularities are compensated by the modified geometrical optics (GO) field.

Performing the integration expressed in (1), we obtain the time-domain version of UAT

$$\overset{+}{\mathbf{E}}^T(\mathbf{r},t) = \overset{+}{\mathbf{E}}^G(\mathbf{r},t) + \overset{+}{\mathbf{E}}^d(\mathbf{r},t) \tag{11}$$

where the first term of (11) corresponds to the modified GO field and is expressed by

$$\overset{+}{\mathbf{E}}^G(\mathbf{r},t) = \frac{1}{2}\overset{+}{f}^i(t) * \overset{+}{\mathbf{E}}^i(\mathbf{r},t) + \frac{1}{2}\overset{+}{f}^r(t) * \overset{+}{\mathbf{E}}^r(\mathbf{r},t). \tag{12}$$

The vectors $\overset{+}{\mathbf{E}}^{i,r}(\mathbf{r},t)$ are respectively the direct [6] and reflected fields [8] (considering that only one of the wedge faces is illuminated). The signal $*$ denotes a time convolution, and the transition functions are

$$\overset{+}{f}^{i,r}(t) = \frac{1}{2\pi\sqrt{-j\tau^{i,r}}\left(\sqrt{-j\tau^{i,r}}+C^{i,r}\sqrt{j}\right)} - \frac{1}{2\pi C^{i,r}\sqrt{\tau^{i,r}}} \tag{13}$$

with

$$\tau^{i,r} = t - (C^{i,r})^2. \tag{14}$$

The diffracted field corresponding to the second term of (11) is expressed as

$$\overset{+}{\mathbf{E}}^d(\mathbf{r},t) = \frac{1}{2}\left[\overset{+}{d}_s\left(t-\frac{s^d}{c}\right) * \overset{+}{E}^i_{\beta_0'}\left(Q_E, t-\frac{s^i(Q_E)}{c}\right)\hat{\boldsymbol{\beta}} + \overset{+}{d}^h\left(t-\frac{s^d}{c}\right) * \overset{+}{E}^i_{\phi_0'}\left(Q_E, t-\frac{s^i(Q_E)}{c}\right)\hat{\boldsymbol{\phi}}\right] A_d(s^d) \tag{15}$$

where the distances $s^{i,d}$ are shown in Fig. 1. The time-domain diffraction coefficients are given by

$$\overset{+}{d}^{s,h}(t) = -\frac{\sqrt{\frac{c}{\pi t}}}{2n\sqrt{2\pi}\sin\beta_0}\sum_{m=1}^{4}K_m^{s,h}. \tag{16}$$

Considering observation points located far away from the shadow boundaries, for which $|C^{i,r}|$ are large numbers, and early instants of wavefront arrival, corresponding to high frequency spectra and where $t \ll (C^{i,r})^2$, the binomial expansion can be applied to $\sqrt{\tau^{i,r}}$

$$\sqrt{\tau^{i,r}} = \sqrt{t-(C^{i,r})^2} \simeq -j|C^{i,r}| + j\frac{t}{2|C^{i,r}|} \tag{17}$$

and the first term of (13) can be expressed as

$$\begin{aligned}&\left[2\pi\sqrt{-j\tau^{i,r}}(\sqrt{-j\tau^{i,r}}+C^{i,r}\sqrt{j})\right]^{-1}\\ &\simeq \frac{j}{2\pi}\left\{(C^{i,r})^2\left[\mathrm{sgn}(-C^{i,r})-1\right]+\frac{t}{2}\right\}^{-1}\\ &= \frac{j}{\pi t}U(-\epsilon^{i,r}),\ \text{for } t \ll (C^{i,r})^2\end{aligned} \tag{18}$$

where $U(\cdot)$ is the unit step function, to yield

$$\overset{+}{f}^{i,r}(t) \approx \overset{+}{\delta}(t)U(-\epsilon^{i,r}) + \frac{1}{2\pi C^{i,r}\sqrt{\tau^{i,r}}}, t \ll (C^{i,r})^2 \tag{19}$$

where the analytic Dirac delta function is given by [8]

$$\overset{+}{\delta}(t) = \frac{j}{\pi t}, \qquad \Im m(t) > 0. \tag{20}$$

Still, the following limit:

$$\lim_{|C^{i,r}| \to \infty} \overset{+}{f}^{i,r}(t) = \overset{+}{\delta}(t) U(-\epsilon^{i,r}) \tag{21}$$

applies, and from (12) and the analytic Dirac delta properties, it follows that [8]

$$\lim_{|C^{i,r}| \to \infty} \overset{+}{\mathbf{E}}^G(\mathbf{r}, t) = \overset{+}{\mathbf{E}}^i(\mathbf{r}, t) U(-\epsilon^i) + \overset{+}{\mathbf{E}}^r(\mathbf{r}, t) U(-\epsilon^r) \tag{22}$$

showing that the TD-UAT formulation for observation points far away from shadow boundaries reduces to a time-domain version of Keller's GTD, exactly as derived by Veruttipong [15] and obtained by Pathak [8] by applying appropriate limits to the TD-UTD formulation.

The desired expression for the total field is then given by the real part of the analytical signal representation in (11)

$$\mathbf{E}^T(\mathbf{r}, t) = \Re e \left[\overset{+}{\mathbf{E}}^T(\mathbf{r}, t) \right], \qquad \Im m(t) = 0. \tag{23}$$

The geometrical parameters involved in the calculation of the scattered field such as spread factors of fields, ray-fixed coordinates, and vector components are calculated as in the frequency domain [14].

III. Numerical Results

Fig. 1 shows the geometry of a straight wedge illuminated by an electrical dipole with an arbitrary orientation in the plane xy. The results presented here were obtained for a dipole located at coordinates $(r' = 100 \text{ m}, \theta' = 90^\circ, \phi' = 45^\circ)$, which corresponds to a normal incidence, with $\beta_0 = 90^\circ$ and $s^i(Q_E) = 100$ m, and for a wedge with $\alpha = 30^\circ$.

The accuracy of the TD-UAT in the wedge response to a realistic excitation (with finite energy) was determined by the response to a dipole-radiated pulse with power spectrum given by

$$P(\omega) = C_0 \left(1 - e^{-|\omega| S} \right) e^{-|\omega| S} \tag{24}$$

where C_0 is a real constant and

$$S = \frac{0.6931}{2\pi f_c} \tag{25}$$

where f_c is the central frequency of the spectrum. It can be shown that this pulse has an analytical representation given by [8], [10]

$$\overset{+}{p}(t) = C_0 \sum_{m=1}^{2} A_m^+ \delta(t + jT_m) \tag{26}$$

where $A_m = \{1/2, -(1/2)\}$ and $T_m = \{S, 4S\}$. This pulse makes the time convolutions in (12) and (15) easy to perform. Fig. 2 shows the TD-UAT response of the wedge to a pulse $P(\omega)$

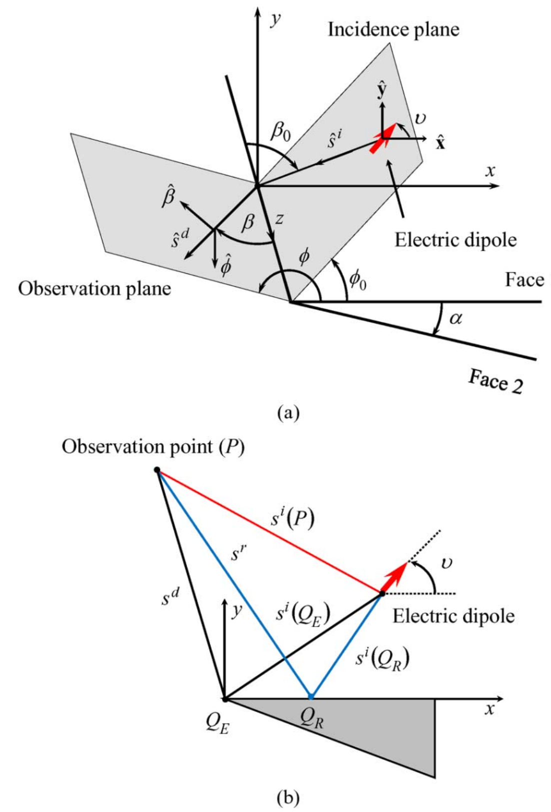

Fig. 1. Geometry of the problem of EM scattering by a PEC wedge. (a) 3-D view. (b) Side view.

with $f_c = 5$ GHz. The scattered field was determined for observation points located at $(r = 100 \text{ m}, \theta = 90^\circ, \phi = 224^\circ)$ [Fig. 2(a)] and $(r = 100 \text{ m}, \theta = 90^\circ, \phi = 230^\circ)$ [Fig. 2(b)], corresponding to $\beta = 90^\circ$ and $s^d = 100$ m. The scattered field for the latter observation point is formed by the merge of direct and edge-diffracted fields. The uniformity of the TD-UAT in the transition regions can be observed. For the former observation point, only the edge-diffracted field exists and the influence of the edge illumination taper is perceived by an enhanced difference between TD-UAT and TD-UTD results, while by adding the slope diffraction terms to the TD-UTD formulation, results tend to coalesce, stressing the equivalent efficiency of both techniques when all relevant diffraction mechanisms are properly taken into account. These results are compared to those obtained from an implementation of the TD-UTD with slope difraction terms [9], with a good agreement between them. The results in Fig. 2(b) were obtained for a dipole with $\nu = 230^\circ$, which poses a slow spatial variation for the incident fields at the point Q_E, therefore implying a closer agreement between TD-UAT results and those from both TD-UTD versions.

The frequency-domain UAT encompasses higher-order diffraction effects. As expected, its time-domain counterpart has the same behavior as can be observed in the results for the wedge scattered field when the dipole is pointing to the edge $(\nu = 225^\circ)$ in Fig. 3, highlighting the role of the edge illumination taper. These results are compared to those obtained by the solution of the TD-UTD in the cases where the slope diffraction terms are not used and when they are. When the dipole is pointed to the edge, the incident field at the diffraction point is zero, and the diffracted-field component obtained by

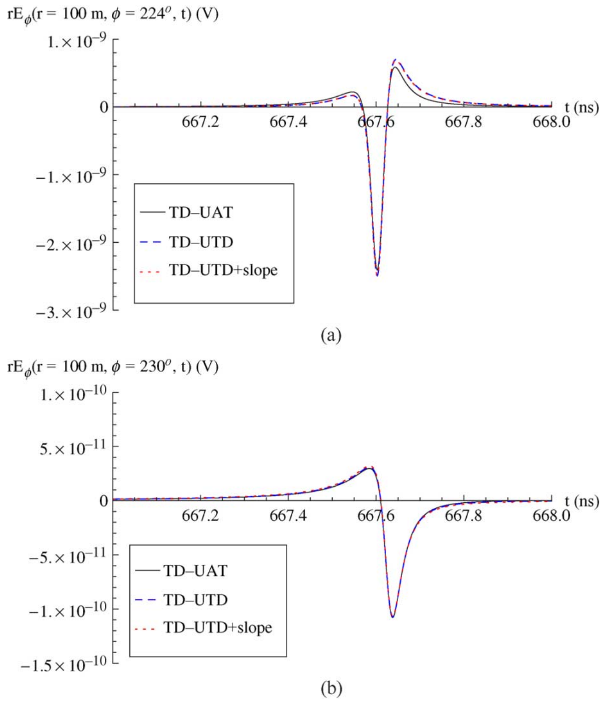

Fig. 2. Scattered field for $\nu = 230^\circ$. (a) $\phi = 224^\circ$. (b) $\phi = 230^\circ$.

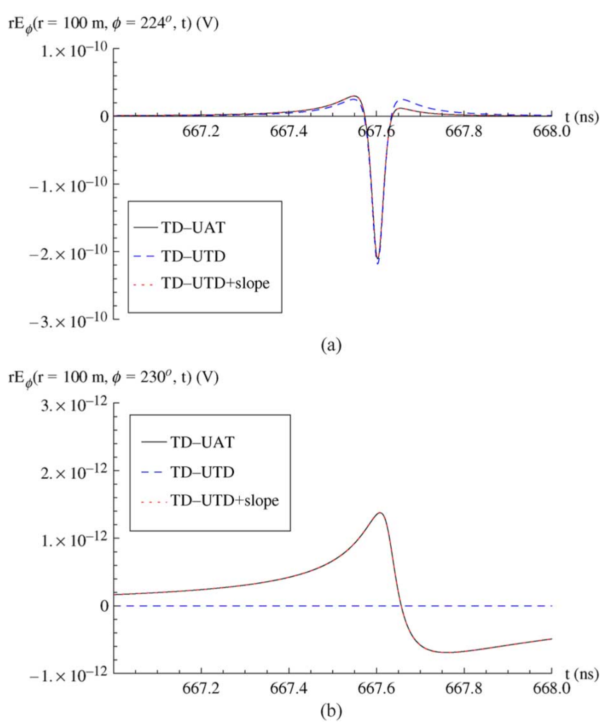

Fig. 3. Scattered field for $\nu = 225^\circ$. (a) $\phi = 224^\circ$. (b) $\phi = 230^\circ$.

the TD-UTD without the slope diffraction terms must be equal to zero, as depicted in Fig. 3(b).

IV. Conclusion

In this letter, a time-domain version of the uniform asymptotic theory of diffraction (TD-UAT) was introduced and applied to the scattering by a PEC wedge of a finite energy pulse radiated by an arbitrarily oriented dipole source. Results ascertained its applicability and evidenced its usefulness, when compared to those of a TD-UTD implementation, for certain observation and edge illumination aspects. While any limitation of the TD-UAT, inhibitory of its implementation in connection with diffraction analyses of pulsed fields, is unknown to the authors, its application to the evaluation of late time responses will most certainly abide by the same constraints of its corresponding (lower frequency) spectra.

The extension of the TD-UAT formulation for arbitrarily shaped (perfectly conducting) surfaces with edges finds its application in the analysis of pulse-excited reflector antennas, and future communications will address its implementation in comparison to alternative methods such as the time-domain versions of EM fields obtained by integration of PO induced currents and the exploration of the role of equivalent edge and fringe-wave currents.

References

[1] E. Heyman and L. B. Felsen, "Weakly dispersive spectral theory of transients, part I: Formulation and interpretation," *IEEE Trans. Antennas Propag.*, vol. AP-35, no. 1, pp. 80–86, Jan. 1987.

[2] E. Heyman and L. B. Felsen, "Weakly dispersive spectral theory of transients, part II: Evaluation of the spectral integral," *IEEE Trans. Antennas Propag.*, vol. AP-35, no. 5, pp. 574–580, May 1987.

[3] E. Heyman and L. B. Felsen, "Weakly dispersive spectral theory of transients, part III: Applications," *IEEE Trans. Antennas Propag.*, vol. AP-35, no. 11, pp. 1258–1266, Nov. 1987.

[4] R. Ianconescu and E. Heyman, "Pulse field diffraction by a perfectly conducting wedge: A spectral theory of transients analysis," *IEEE Trans. Antennas Propag.*, vol. 42, no. 6, pp. 781–789, Jun. 1994.

[5] L. B. Felsen and N. Marcuvitz*, Radiation and Scattering of Waves*. Englewood Cliffs, NJ: Prentice-Hall, 1973.

[6] L. B. Felsen, "Diffraction of a pulse field from an arbitrarily oriented electric or magnetic dipole by a perfectly conducting wedge," *SIAM J. Appl. Math.*, vol. 26, no. 2, pp. 306–312, Mar. 1974.

[7] K.-K. Chan and L. B. Felsen, "The pulsed field due to an electric dipole in the presence of a perfectly conducting wedge," *IEEE Trans. Antennas Propag.*, vol. AP-25, no. 3, pp. 420–423, May 1977.

[8] P. R. Rousseau and P. H. Pathak, "Time-domain uniform geometrical theory of diffraction for a curved wedge," *IEEE Trans. Antennas Propag.*, vol. 43, no. 12, pp. 1375–1382, Dec. 1995.

[9] P. R. Rousseau and P. H. Pathak, "TD-UTD slope diffraction for a perfectly conducting curved wedge," in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, Newport Beach, CA, Jun. 1995, pp. 856–859.

[10] P. R. Rosseau, P. H. Pathak, and H.-T. Chou, "A time domain formulation of the uniform geometrical theory of diffraction for scattering from a smooth convex surface," *IEEE Trans. Antennas Propag.*, vol. 55, no. 6, pp. 1522–1534, Jun. 2007.

[11] E.-Y. Sun and W. V. T. Rusch, "Time-domain physical-optics," *IEEE Trans. Antennas Propag.*, vol. 42, no. 1, pp. 9–15, Jan. 1994.

[12] C. G. Rego, S. T. M. Gonçalves, and F. J. S. Moreira, "High-frequency asymptotic formulation for prompt response of parabolic reflector antennas," *Int. J. Electron. Commun.*, vol. 64, pp. 36–46, Jan. 2010.

[13] C. G. Rego and F. J. V. Hasselmann, "Equivalent edge currents for the analysis of pulse-excited reflector antennas (in Portuguese)," *Brazil. Telecommun. Soc. Mag.*, vol. 21, no. 1, pp. 1–15, 2006.

[14] S.-W. Lee and G. A. Deschamps, "A uniform asymptotic theory of electromagnetic diffraction by a curved wedge," *IEEE Trans. Antennas Propag.*, vol. 24, no. 1, pp. 25–34, Jan. 1976.

[15] T. W. Veruttipong, "Time domain version of the uniform GTD," *IEEE Trans. Antennas Propag.*, vol. 38, no. 11, pp. 1757–1764, Nov. 1990.