2. A Deterministic Neoclassical Growth Model Extended for Productive Public Capital, Frictions on Capital Accumulation and Indivisible Labor Supply

In this section, we outline an otherwise standard neoclassical growth model extended for a time-to-build process for public capital, variable capital utilization rate, adjustment costs in private investment and indivisible labor supply. We suppose an economy, denoted by local, which may receive outside transfers to finance public investment conducted by the local public sector. These transfers should be interpreted as federal grants received by state governments. According to the previous discussion in Section 2, this approach is consistent with the methodology applied by Leduc and Wilson (2012) to the Federal-Aid Highway Program in United States.

2.1. Households

There is a continuum of identical infinitely lived households who lie in the interval [0,1]. In the model, the individual chooses lotteries instead of hours, and the lottery determines whether or not he works, as in Hansen (1985). Once employed, the agent works a fixed amount of hours, denoted by $\bar{l}$.[3] In such a framework, each household maximizes her lifetime utility given by $\sum_{t=0}^{\infty} \beta^t U(c_t, p_t)$, where β is the subjective discount factor, common to all agents, c_t is private consumption and p_t is the probability of working.[4] We suppose an expected instantaneous utility function of the form

$$U(c_t, p_t) = \log(c_t) - p_t \varphi \frac{\bar{l}^{\xi}}{\xi}, \quad (1)$$

[3] As explained by Hansen (1985), introducing a lottery in such a way causes the consumption possibilities set to be convex, so that the competitive equilibrium can be determined by the solution of a concave programming problem. The lottery should be understood as a contract between the household and the representative firm, in which the agent is completely insured but commited to work a fixed number $\bar{l}$ of hours with probability p_t.

[4] The lotteries (and, consequently, the probability of being employed) play no role in the quantitative predictions of the model. Their purpose is only to make our dynamics of hours worked consistent with an interpretation of variations in the extensive margin of labor supply.

where we normalize, without loss of generality, $\bar{l} = 1$.[5]

Since all agents are identical, they will choose exactly the same lottery, which is equal to the fraction of households actually employed, p_t. In this case, aggregate hours are given by

$$l_t = p_t \bar{l} = p_t.$$

Therefore, we can write the representative household`s instantaneous utility function as

$$U(c_t, l_t) = \log(c_t) - \theta l_t, \tag{2}$$

with $\theta \equiv \varphi/\xi$.[6]

The model also incorporates two frictions on the process of capital accumulation. First, we assume that the representative household can choose the utilization rate, u_t, and it influences the degree at which the capital stock depreciates, according to the following functional form

$$\delta(u_t) = \delta \exp\{\omega(u_t - 1)\}, \tag{3}$$

where $\omega \equiv (1/\beta - 1)(1/\delta) + 1$ is set by steady state restrictions.[7]

Second, we introduce adjustment costs in private investment. We follow the functional form adopted by Jermann and Quadrini (2009)

$$k_{t+1} = (1 - \delta(u_t))k_t + \left[\frac{\eta_1 \left(\frac{i_t}{k_t}\right)^{1-\nu}}{1-\nu} + \eta_2\right] k_t, \tag{4}$$

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[5] It is straightforward to show that the consumption level of a household, c_t, is the same when she is employed and unemployed.

[6] The instantaneous utility function derived for the representative agent implies an infinite elasticity of substitution between hours in different periods, no matter how small it is for each household populating the economy. As pointed out by Hansen (1985), this result remains valid as long as the utility function is separable across time.

[7] The steady state relationship is obtained by the optimality condition concerning u_t (where $u = 1$ in steady state) showed in the Appendix A.

where ν measures the degree of the cost to investment imposed on the capital accumulation process. If we impose $\nu = 0$, the standard law of movement for the capital stock is obtained. The parameters η_1 and η_2 are determined by the steady state conditions applied to the model.[8]

At this point, we can state the representative household problem:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{5}$$

subject to

$$c_t + i_t \leq w_t l_t + r_t u_t k_t + T_t \text{ and (4),}$$

where T_t are lump-sum taxes and $U(c_t, l_t)$ is given by (2).

2.2. Firms

We assume a representative firm which rents capital and hires a fraction $l_t = p_t$ of households. It uses a Cobb-Douglas technology extended for the presence of productive public capital:

$$y_t = (u_t k_t)^{\alpha} {l_t}^{1-\alpha} (K_t^G)^{\alpha^G}, \tag{6}$$

where K_t^G is the public capital stock which works as an externality over the output production, whose productivity is measured by α^G. This way of modeling productive public capital is usual in the literature, as in Barro (1990) and Baxter and King (1993).

The firm's optimality conditions imply that in equilibrium

$$r_t = \alpha \frac{y_t}{u_t k_t}; \; w_t = (1-\alpha)\frac{y_t}{l_t}. \tag{7}$$

[8] As pointed out by Jermann and Quadrini (2009), η_1 and η_2 are set by two steady state conditions. First, we impose that the steady state depreciation rate is equal to δ, and, second, that $\partial k_{t+1}/\partial i_t$ = 1. The latter condition implies that the Tobin`s q is equal to 1 in steady state.

2.3. Government

We follow the same framework adopted by Leeper, et al. (2010) – also followed by Leduc and Wilson – to model the infrastructure spending. Let A_t denote the outside transfers received by the local government to invest in public capital. Once a shock hits the value of outside transfers, it evolves according to

$$A_t = (1 - \rho_A)A + \rho_A A_{t-1} + \in_t^A, \tag{8}$$

where A are the steady state public consumption and ϵ_A is an unanticipated shock in the current value of outside transfers.

Turning to local fiscal policy, we assume that there are implementation delays in public investment according to

$$I_t^G = \sum_{n=0}^{N} \phi_n A_{t-n}, \tag{9}$$

where $\sum_{n=0}^{N} \phi_n = 1$.[9] The spending rates $\{\phi_n\}_{n=0}^{N}$ determine the rate at which the outside transfers are spent by the local government over time.

In the time-to-build process for infrastructure projects, the outside transfers received by the local government turn into public capital only *M* periods later:

$$K_t^G = (1 - \delta_G)K_{t-1}^G + I_{t-M}^G. \tag{10}$$

An important feature in the model refers to how infrastructure spending is financed. As previously discussed in Section2, Leduc and Wilson, in applying their estimation technique, control for aggregate time fixed effects. Thus, any potential negative wealth effect associated to current and future federal tax increases effectively disappears in the estimated IRFs. Yet, the impact of local fiscal policy remains unaltered in the estimation procedure. The methodology implies that projects may be windfall-financed (i.e. they may be fully financed by federal grants) as well as partially financed by the local government through

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[9] Since $\sum_{n=0}^{N-1} \phi_n = 1$, $I^G \equiv A$ in steady state, where A is the steady state level of outside transfers.

current and future taxes. Actually, the authors point out that federal-aid highway expenditures are part windfall-financed, to the extent that reimbursable outlays need to be accompanied by nonreimbursable state spending on police services, traffic control and other related services.

In this sense, we also allow for the presence of flypaper effects in local public consumption, which responds to infrastructure spending:

$$C_t^G = \tilde{C}^G + \gamma I_t^G, \tag{11}$$

where $\tilde{C}^G \equiv C^G - \gamma I^G$. C^G and I^G are the steady state local public consumption and local government investment, respectively. Together with the previous discussion, the presence of a flypaper effect is also supported by the very high GDP multipliers implied by shocks to highway spending, found by Leduc and Wilson.[10] The above specification should be viewed as a reduced form for the flypaper effect on public consumption, since, conceptually, we should relate C_t^G directly with A_{t-n}, n = 0, 1, 2… In this case, the relevant flypaper effect parameters are given by $\{\gamma\phi_n\}_{n=0}^N$.

In fact, we can see that

$$C_t^G = \tilde{C}^G + \gamma I_t^G = \tilde{C}^G + \sum_{n=0}^{N} \gamma_n A_{t-n}, \tag{12}$$

where $\gamma_n \equiv \gamma\phi_n$, n = 0, 1, …, N.

The literature on the flypaper effect is relatively large, and the majority of studies tend to confirm its presence within a broad range of public expenditures.[11] In the model considered here, the flypaper effect plays no significant role, except to increase the tax burden on households. In fact, in the goods market clearing condition, after substituting the equation (11), we have:

[10] Leduc and Wilson (2012) find multipliers on impact lying between 1.4 and 3.4, whereas the peak multipliers stay between 3.0 and 7.8. Naturally, the broader is the concept of highway spending considered, the lower is the implied GDP multiplier.

[11] In the recent literature on the flypaper effect, we can cite Baicker (2005), Evans and Owens (2007), Singhal (2008) and Feiveson (2011) as papers that find strong evidences of flypaper effects. On the other hand, Knight (2002) and Gordon (2004) find evidence contrary to it.

$$y_t = c_t + i_t + C^G + I^G + (\pi + \gamma)(I_t^G - I^G), \tag{13}$$

where π measures the portion of new public capital that is financed by the local government. Thus, imposing a flypaper effect on public consumption is equivalent to assuming that the local government, in its absence, finances $\pi + \gamma$ of highway spending. In the case $\pi = 0$ ($\pi = 1$), we have windfall-financed (fully tax financed) infrastructure expenditures.

2.4. Calibration Procedures

In calibrating parameters, especially those associated to fiscal policy and technology concerning public capital, we try, as far as possible, to follow the parameterization adopted by Leduc and Wilson (2012) in their model. A restricted calibration allows to assessing the quantitative performance of the model, since we reduce the number of free parameters.

Concerning the parameters of preferences, we set the subjective discount factor, β, to 0.97. On the other hand, it is worth reminding that the Frisch labor elasticity of the representative household is infinite, no matter how large or small the Frisch labor elasticity, $1/(\xi - 1)$, is for each household populating the economy. Thus, we normalize $\xi = 1$, and determine φ ($\equiv \theta$) to imply a steady state probability of working, p ($= l$), of 0.70.

Turning to technological parameters, the adjustment costs in investment, ν, are set to 0.01 in the baseline calibration to match the initial empirical response of employment. Other values are considered, depending on the degree of lump-sum taxes that are used to finance public investment. The steady state depreciation rate of the capital stock, δ, is chosen to be 0.10. Turning to capital income share in output, α, we set a value of 0.36. A crucial parameter in the model is the elasticity of public capital to output, α^G. Importantly, the larger is α^G, the deeper are the implied theoretical recessions. In fact, a higher value of α^G increases the (positive) wealth and intertemporal substitution effects on labor supply and investment decisions respectively, since agents expect higher marginal productivities once the new public capital is installed. Unfortunately, the empirical evidence regarding the productivity of public capital is inconclusive, and findings range from slightly negative to large positive impacts.[12] Thus, we pick a conservative value of 0.10, the same adopted by Leduc and Wilson.[13] We also follow them in calibrating the persistence of the shocks to outside transfers, ρ_A, and the spending rates, $\{\phi_n\}_{n=0}^{N}$. We set ρ_A = 0.27 and (ϕ_0, ϕ_1) = (0.70, 0.30). The latter calibration implies that 70% of outside transfers are spent in the current year and 30% in the next. The depreciation rate of public capital, δ_G, and the time-to-build process, M, are calibrated according to Leduc and Wilson values of 0.10 and 4, respectively.

Finally, the parameter linking local public consumption to local public investment, γ, is set to 0.30 in the baseline calibration. Such value implies small flypaper effects in the magnitude of \$0.30. In fact, a one-dollar increase in current outside transfers, A_t, leads to a subsequent increase in local public consumption of $\gamma_0 = \phi_0 \times \gamma$ = (0.70)(0.30) = 0.21 cents. In the same way, a one-dollar increment in lagged transfers, A_{t-1}, leads to an increase in local expenditures of $\gamma_1 = \phi_1 \times \gamma$ = (0.30)(0.30) = 0.09 cents. In order to check the sensitivity of our results, we also report charts when $\gamma = 0$.

In the analysis, the theoretical IRFs are obtained through a shooting-algorithm method, and we treat the empirical ones as an approximation of a broad range of shocks to public infrastructure expenditures. In this way, we set the total government spending-output ratio to 18%, according to the United States national level. The public consumption (investment) corresponds to 14% (4%) of output. Turning to the degree of non-distortionary taxation, we follow institutional characteristics of the Federal-Aid Highway Program, allowing that the local government finances up to 20% of infrastructure spending ($\pi \in \{0, 0.20\}$). In the baseline calibration, we set π = 0.20, and check results imposing π = 0.[14] Actually, since the federal government reimburses states up to 90% of the cost of eligible projects, states finance at least 10% of highway expenditures. Thus, imposing $\pi = 0$ refers to an extreme and unrealistic scenario. In Table 1, we provide a summary of the parameterization.

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[12] Holtz-Eakin (1994), Evans and Karras (1994) and Kamps (2004) report negative to muted productivity effects of public capital. On the other hand, Nadiri and Mamuneas (1994) find positive impacts of infrastructure on the private sector.

[13] Leduc and Wilson (2012) follow Baxter and King (1993) and Leeper, Walker and Yang (2010).

[14] Assuming γ = 0.30 together with π = 0.20 is equivalent to suppose that the local government finances 50% of highway spending in the absence of flypaper effects, according to the goods market clearing condition.

Table 1: Baseline Calibration

Calibrated Parameters					
Preferences		Technology		Fiscal Policy	
β	0.97	α	0.36	I^G/y	0.04
θ	1.68	ν	0.01	C^G/y	0.14
				γ	0.30
Leduc and Wilson (2012) Calibration					
		Technology		Fiscal Policy	
		α^G	0.10	ϕ_0	0.70
		δ	0.10	ϕ_1	0.30
		δ_G	0.10	π	0.20
		M	4		

Therefore, in the quantitative analysis that follows, we have only two free parameters (the adjustment costs in investment, measured by ν, and the degree of flypaper effects, γ). To the extent that we have a very simple neoclassical growth model within a restricted parameterization, we believe that our approach is quite parsimonious.