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Effective Resource Allocation for Planning and Control Project Portfolios Under Uncertainty: A Robust Optimization Approach

Tese de Doutorado

Thesis presented to the Programa de Pós-Graduação em Informática of the Departamento de Informática, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Ciências - Informática

Advisor : Prof. Marcus Vinicius Soledade Poggi de Aragão
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Abstract

Mendes, Carlos Raoni de Alencar; Aragão, Marcus Vinicius Soledade Poggi de (advisor); Flach, Bruno da Costa (co-advisor). **Effective Resource Allocation for Planning and Control Project Portfolios Under Uncertainty: A Robust Optimization Approach**. Rio de Janeiro, 2017. 112p. Tese de Doutorado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

Planning and controlling complex project portfolios is a challenging task. These portfolios are subject to a number of potential risk sources coupled with resource constraints, intricate precedence relationships, and penalties for project delays. For this reason, it is fundamental that optimal strategies for the allocation of the available resources are constantly adopted by the decision makers to ensure that their projects are completed within limits of time and cost. Moreover, the uncertainty that affects these projects has to be taken into account for effective resource allocation decisions. Within this context, this work proposes a robust optimization-based methodology for planning and controlling project portfolios under uncertainty. The method combines models and algorithms for multiple resource allocation problems under the same robust optimization framework. In this approach, the uncertainty environment is modeled as an adversary that selects the worst-case combination of risks for any decision maker's actions. Subsequently, the main goal of the decision maker is to determine optimal resource allocation plans for minimizing a particular objective subject to the assumption that the adversary's worst-combination of risks will materialize. The approach also provides a way to control the degree of conservatism of the solutions. For each studied problem, a solution strategy is developed through a reformulation scheme from a compact min-max formulation to a cut-generation algorithm. Several computational experiments are conducted, providing key insights that drive the design of the referred portfolio planning and control methodology. The ineffectiveness of traditional critical path analysis under worst-case realizations of uncertain activities' durations and the importance of taking integrated resource allocation decisions in the context of project portfolios, are examples of the key findings of the experiments. The application of the methodology is demonstrated in a case study of a portfolio aimed at the construction of two refineries. This example presents the capabilities of the developed techniques in a practical context.

Keywords

Project Portfolios; Resource Allocation; Stochastic Programming; Robust Optimization; Decision-Dependent Uncertainty; Project Management; Risk Analysis;

Resumo

Mendes, Carlos Raoni de Alencar; Aragão, Marcus Vinicius Soledade Poggi de; Flach, Bruno da Costa. **Alocação Efetiva de Recursos para Planejamento e Controle de Portfolios de Projetos sob Incerteza: Uma Abordagem de Otimização Robusta**. Rio de Janeiro, 2017. 112p. Tese de Doutorado — Departamento de Informática, Pontifícia Universidade Católica do Rio de Janeiro.

O planejamento e controle de portfolios de projeto é uma tarefa desafiadora. Eles estão sujeitos a múltiplos riscos, restrições de recursos, relações de precedências e penalidades por atrasos de projetos. É fundamental desenvolver estratégias efetivas de alocação dos recursos disponíveis de forma a garantir que estes projetos sejam concluídos dentro dos limites de tempo e custo. Um fator crucial que deve ser levado em consideração ao tomar estas decisões é o gerenciamento das incertezas associadas a execução dos projetos. Neste contexto, este trabalho propõe uma metodologia baseada em otimização robusta para planejamento e controle de portfolios de projeto sob incerteza. Este método combina modelos e algoritmos desenvolvidos para diferentes problemas de alocação de recursos para os quais foi aplicada a mesma abordagem de otimização robusta. Nela, a incerteza é modelada como um adversário capaz de materializar a combinação de riscos de pior caso que maximiza o impacto no(s) projeto(s) para qualquer plano de alocação de recursos. Nos problemas estudados o tomador de decisão tem então que determinar a alocação ótima de recursos que minimiza um objetivo particular assumindo que a combinação de riscos de pior caso irá se materializar. A abordagem também provê um mecanismo para controle do grau de conservadorismo das soluções robustas. Para cada problema modelado, uma estratégia de solução é desenvolvida através de um esquema de reformulação que parte de uma formulação Min-Max compacta e termina em um algoritmo de geração de cortes. Diversos experimentos computacionais foram executados, provendo importantes conclusões que direcionaram o desenvolvimento da metodologia de controle e planejamento de portfolios. A importância de se desenvolver planos de alocação de recursos de forma integrada no contexto de tomada de decisão em portfolios de projetos e a falta de efetividade do método tradicional de análise de caminhos críticos no contexto de cenários de pior caso para as durações das atividades, são importantes exemplos das conclusões obtidas pelos experimentos. A aplicação da metodologia foi demonstrada em um caso de estudo que contempla um portfolio para construção de duas refinarias. O referido exemplo demonstrou o potencial do uso prático dos métodos propostos neste trabalho.

Palavras-chave

Portfolios de Projetos; Alocação de Recursos; Otimização Estocástica; Otimização Robusta; Incerteza Dependente de Decisão; Gerenciamento de Projetos; Análise de Riscos;

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Quem já te viu, ó não te esquece mais!

Martins D'Alvarez, *trecho do hino da cidade de Crato, Ceará.*

1
Introduction

1.1
Motivation and Objective

Construction of oil platforms, refineries, mining storage facilities, and thermal and hydroelectric power plants are examples of huge projects that are often part of a company's and government's strategic plans. Planning, monitoring, and managing such complex projects or their portfolio is a challenging task as it involves a number of potential risk sources coupled with resource constraints, intricate precedence relationships, and penalties for delays of key milestones.

In 2012, the consulting firm A.T. Kearney conducted a cross-industry survey to evaluate the performance of key projects in industries such as oil and gas, mining, chemicals, and telecommunications (Kearney, 2012). According to the survey, 63% of the projects were over budget, with 14% having more than 10% of cost overrun. Kearney also found that 75% of them were behind schedule (or delayed), with 21% having more than 10% of total delay. Figure 1.1 presents the detailed results of budget and schedule performances by percentage levels of overrun. In 2016, the company conducted a new survey that obtained similar results, revealing that there were no performance improvements during the four years interval (Kearney, 2016). In the context of projects costing over US$1 billion, the reality is even worse, approximately nine out of ten have cost overruns, with the same statistic holding for delays (Flyvbjerg, 2014). These findings illustrate the difficulty of properly managing important projects of different industries.

The economic feasibility of a company's long-term strategic plan might be strongly connected to a successful project (or portfolio) delivery; for instance, delays could cause a late arrival on a potential strategic market and cost overruns could damage its investment capacity. Hence, these cascading impacts threaten a company's financial stability and may seriously affect its ability to honor commitments and engage in revenue-generating activities (Flyvbjerg, 2014). Given the circumstances, it is fundamental that the staff

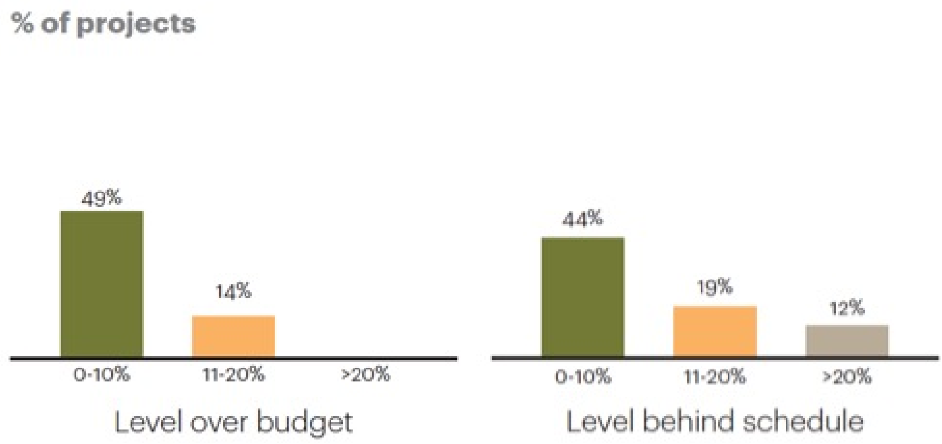


Figure 1.1: Percentage of over-budget and delayed projects by level from Kearney (2012).

responsible for managing these key projects are supported by decision-making methods and tools that aid them to handle all the complexities involved in the execution and, therefore, increase the chance of projects being delivered within limits of time, cost, and scope (Öncü Hazır, 2015). As projects are inherently uncertain (Hans et al., 2007), effective decision-making support methods have to provide a way to properly represent and deal with this uncertainty. Within this context, decision makers have to effectively manage the mostly limited available resources to control the project (or portfolio) execution under uncertain conditions. The term "resource allocation" is being used here in a broad perspective – from the assignment of extra workers or the use of more efficient machinery, to improve activity productivity, to the implementation of mitigation plans to minimize potential impacts of uncertain events (or risks).

Operations Research (OR) methods have been commonly applied to model and solve various resource allocation problems in the context of project (or portfolio) management under uncertainty. In Öncü Hazır (2015), authors have reviewed several methods and concluded that there is a theory-practice gap, while academic studies usually investigate closed and less complex problems, managers face multi-dimensional dynamic and open systems. For instance, a common trend is to have organizations managing several projects simultaneously that compete for the available resources. Furthermore, the management of this portfolio of interdependent projects is frequently performed by separating the decision-making process for each individual project, resulting into suboptimal solutions with respect to the organization goals. In this case, the resource allocation strategies should consider all projects in an integrated fashion. In Kearney (2016), the authors have advocated that focusing on strengthening portfolio management and investment decision making is one of the fundamental actions to improve project's performance. Öncü Hazır (2015) and Hans et al. (2007) have also highlighted the importance of developing multi-project or portfolio management methods. As detailed in Chapter 3, the important academic literature of multi-mode resource allocation and project scheduling have almost no intersection with the works on practical risk analysis or management. While both fields have their own importance, methods that combine key elements of these theories are still scarce. Herroelen (2005) has suggested the integration of scheduling theory and risk analysis tools into the current project management practice as one of the research frontiers to fill (or reduce) the referred theory-practice gap.

Within this context, the main objective of this work is to develop an effective decision support method for planning and controlling a portfolio of interdependent projects subject to uncertainty. Taking into account the previous discussion, the following requirements are considered in the method development:

– supporting an uncertainty model inspired on practical risk analysis methods;
– supporting budget-constrained resource allocation distributed between strategies to accelerate activities (mode selection or activity crashing) and strategies to alleviate the potential impact of risks (risk mitigation);
– modeling projects' due dates (or deadlines) and corresponding penalties for delays;
– taking into account precedence constraints between activities of different projects, creating an integrated portfolio network; and
– devising effective integrated resource allocation plans throughout the execution of portfolio projects, protecting them against uncertainty disruptions.

1.2 Methodology

In recent years, the robust optimization paradigm for decision under uncertainty has been gaining the attention of the scientific community (Gabrel et al., 2014). This paradigm contemplates a series of techniques that allow decision makers to protect their decisions against parameter ambiguity and stochastic uncertainty (Ben-Tal et al., 2009). Given these characteristics, it is a well-suited paradigm for the development of the described decision support method subject of this work. Therefore, we proposed a robust optimization approach to handle the inherent uncertainty of projects' executions. In this approach, the uncertain environment is modeled as an adversary that selects the worst-case (highest impact) combination of risks given the decision maker's actions. Then, the decision maker has to determine optimal resource allocation plans for minimizing a particular objective subject to the assumption that the adversary's worst-case combination of risks will materialize. The worst-case objective value minimization provides performance guarantees that are of great value for project control. The chosen robust strategy is applied to two relevant problems, mapping different decision-making challenges in the development of our target portfolio control method. After proposing models and solution strategies for these problems, we combined these techniques under a continuous management and control framework that fulfills the designed requirements.

The first studied problem aimed to answer the question of determining which activities should be the focus of uncertainty control measures to guarantee that a project finishes on time. This fundamental question is at the heart of traditional and highly adopted project management techniques, such as the critical path method (CPM) (Kelley & Walker, 1959) and the program (or project) evaluation and review technique (PERT) (Fazar, 1959). The main difference in our work is that we approached this problem under a robust optimization framework, developing a new criticality index for the activities. In this problem, for easiness of representation and analysis, we assumed that the uncertainty effects on activities' durations are independent. This is a common representation adopted in the literature (see Hulett (2009)). The conducted computational experiments revealed that, for complex project networks, traditional critical path analysis is not effective under worst-case realizations of the uncertainty, validating our proposed criterion.

The next problem subject of our work modeled the main dimensions of the described method requirements. It studied the problem of determining effective resource allocation plans to minimize the total cost of a portfolio of interdependent multi-mode projects, under the assumption that a worst-case uncertainty set scenario will unfold. In this problem, each project is subject to tardiness penalties, and we adopted a risk-based approach to model the uncertainty; as such, risk events are the source of the uncertainty affecting activities durations and costs. A particular risk could affect multiple activities simultaneously, which provided a way to correlate uncertainty effects on them (Hulett, 2009). We conducted a series of experiments that illustrate the importance of integrated resource allocation decisions when dealing with portfolios of interdependent projects. These experiments also provided evidence of the effectiveness and computational feasibility of the proposed solution algorithm.

While the developed models and algorithms for the two previously described problems have their applications and relevance on their own, we proposed to combine them in a global continuous portfolio control method that was based on robust optimization. In the present research, we present a case study of the method applied to a portfolio aimed at the construction of two twin refineries that were based on the example of Hulett (2009). This case study highlights important capabilities of the developed techniques proposed in this work, such as the models' flexibility and capacity of devising cost-effective resource allocation plans considering the interdependencies at the integrated portfolio network.

1.3 Contributions

In general, the work provides a flexible framework to model complex decision-making realities in the context of project scheduling and risk analysis. In detail, the main contributions are as follows:

- a robust optimization criterion (or index) for criticality of activities.
- a cut-generation solution algorithm for determining the robust criticality of the activities.
- a polynomial-time separation algorithm for the robust criticality cut-generation procedure.
- a portfolio total cost model that jointly considers: (i) budget- constrained resource allocation decisions regarding multiple activities' modes (crashing) and risk-mitigation plans; (ii) the adoption of a risk-based characterization of uncertainty in duration and cost of the activities; (iii) an uncertainty environment modeled as an adversary with a controlled degree of conservatism; and (iv) a portfolio of interdependent projects subject to tardiness penalties.
- a compact bi-level robust optimization formulation of the portfolio total cost problem.
- a reformulation scheme and its resolution algorithm for the portfolio total cost problem.
- a global methodology for portfolio planning and control based on robust optimization.

1.4 Outline

Chapter 2 presents the necessary background of robust optimization that is applied in the proposed approach. Chapter 3 reviews the literature on related problems. Chapter 4 is dedicated to the development of the new activity criticality index based on robust optimization. In Chapter 5, we develop the described robust approach to the problem of minimizing the total cost of a portfolio of interdependent multi-mode projects with a risk-based model for the uncertainty. The developed method has a fine-grained control of the solutions' conservativeness and also accounts for the decision-dependent uncertainty aspect of the problem. Chapter 6 discusses the final global portfolio planning and control methodology. Finally, Chapter 7 presents concluding remarks and future research directions.

2
Robust Optimization

The robust optimization paradigm contemplates a series of techniques that allow decision makers to protect their decisions against parameter ambiguity and stochastic uncertainty (Ben-Tal et al., 2009). A solution of an optimization problem subject to uncertainty is called robust if it provides a guarantee that a pre-defined criterion would be satisfied for all realizations of the uncertain parameters. A common criterion is the guarantee of solution feasibility, which means that the solution remains feasible for any possible realization of the parameters. Another important criterion is a guarantee that the objective value of the solution is at least (or at most) a certain value in all possible realizations. To provide objective value robustness, most techniques adopt the strategy to evaluate the solution using the worst-case realization of the uncertain parameters, that is, the scenario that results in the most unfavorable objective value (Gabrel et al., 2014).

A fundamental question that arises in robust optimization is the over-conservatism of the solutions. Most techniques provide a way of controlling the degree of conservatism of their solutions. Instead of providing a robustness guarantee for all possible scenario realizations, the guarantee is only limited to scenarios contained in the so-called uncertainty set. In this work, the proposed approach adopts a robust worst-case evaluation criterion for scenarios contained in an uncertainty set that could be controlled by parameter calibration. The approach was inspired in the work of Bertsimas & Sim (2004). For an extensive overview of robust optimization literature refer to (Gabrel et al., 2014). Next, we briefly describe two seminal robust optimization methods and then detail Bertsimas and Sim's approach.

The three methods are described in the context of linear optimization. Consider the nominal linear optimization problem:

$$\text{Max} \quad \mathbf{c^T x} \tag{2-1}$$

$$s.t. \quad \mathbf{Ax} \leq \mathbf{b} \tag{2-2}$$

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \tag{2-3}$$

To model the uncertainty, consider a particular row i of the matrix $\mathbf{A}$, J_i represent the set of coefficients in row i that are subject to uncertainty. Each element a_{ij}, $j \in J_i$ is modeled as a symmetric and bounded random variable $\tilde{a}_{ij}$, $j \in J_i$ that takes values in $[a_{ij} - \bar{a}_{ij}, a_{ij} + \bar{a}_{ij}]$. Without loss of generality, the model assumes that the uncertainty only affects elements of the matrix $\mathbf{A}$. To handle the uncertainty in $\mathbf{c}$, one could simply replace the objective function to a maximization of a new variable z, and add the constraint $z - \mathbf{c^T x} \leq 0$.

2.1 The Robust Formulation of Soyster

The work of Soyster (1973) has inaugurated the field of robust optimization. Under the previously detailed uncertainty model, the robust Soyster's formulation is as follows:

$$
\begin{aligned}
\text{Max} \quad & \mathbf{c^T x} && (2\text{-}4)\\
s.t. \quad & \sum_j a_{ij} x_j + \sum_{j \in J_i} \bar{a}_{ij} y_j \leq b_i \qquad \forall i && (2\text{-}5)\\
& -y_j \leq x_j \leq y_j \qquad \forall j && (2\text{-}6)\\
& \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} && (2\text{-}7)\\
& \mathbf{y} \geq \mathbf{0} && (2\text{-}8)
\end{aligned}
$$

The extra y_j variables are equivalent to the module of corresponding x_j variables at the optimal solution. In Soyster (1973), the authors have demonstrated that this simple model guarantees solution feasibility for all realizations of the uncertain parameters a_{ij}, $j \in J_i$. Yet, this guarantee comes at a price, compromising too much of objective value (Ben-Tal et al., 2009). Moreover, a number of combinations of parameter realizations are unlikely to occur, so this full feasibility guarantee could be too conservative in some contexts.

2.2 The Robust Formulation of Ben-Tal and Nemirovski

In the study of Ben-Tal & Nemirovski (2000), the authors have approached the over-conservatism of robust solutions by proposing a method that ensures feasibility only for realizations in the so-called uncertainty set. An input parameter was used to calibrate this uncertainty set controlling the trade-off between feasibility and objective value. The proposed robust formulation of Ben-Tal & Nemirovski (2000) is as follows:

$$\text{Max} \quad \mathbf{c^T x} \tag{2-9}$$

$$s.t. \quad \sum_{j} a_{ij}x_j + \sum_{j \in J_i} \bar{a}_{ij}y_{ij} + \Omega_i \sqrt{\sum_{j \in J_i} \bar{a}_{ij}^2 z_{ij}^2} \leq b_i \qquad \forall i \tag{2-10}$$

$$-y_{ij} \leq x_j - z_{ij} \leq y_{ij} \qquad \forall i, j \in J_i \tag{2-11}$$

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \tag{2-12}$$

$$\mathbf{y} \geq \mathbf{0} \tag{2-13}$$

The authors have demonstrated that the probability of a constraint i being violated is at most $exp(-\Omega_i^2/2)$. Thus, by adjusting Ω parameters, decision makers could calibrate the level of conservatism of the robust solutions. This model is a second-order cone problem, which may impose additional difficulty in solving than the original linear optimization model and is not attractive to robust discrete models (Bertsimas & Sim, 2004).

2.3 The Robust Formulation of Bertsimas and Sim

The work of Bertsimas & Sim (2004) has proposed a robust model that preserves the linearity of the nominal problem and at the same time allows the adjustment of the level of conservatism through parameter calibration. Their method was successfully applied to several problems in different areas (Bertsimas et al., 2011).

Given the described uncertainty model, consider the i-th constraint of the nominal problem $\mathbf{a}_i^T\mathbf{x} \leq b_i$. For every i, a parameter Γ_i that takes values in the interval $[0, |J_i|]$ is defined. The role of Γ_i is to allow the control of the level of conservatism of the method. Intuitively, it is unlikely that all parameters a_{ij} will vary simultaneously due to the uncertainty. Then, the method proposes to protect the solutions against all changes in up to $\lfloor \Gamma_i \rfloor$ of the coefficients and one coefficient a_{ij} changing at most $(\Gamma_i - \lfloor \Gamma_i \rfloor)\bar{a}_{ij}$. To model this robustness criterion, a protection function β_i is defined for each constraint i. Given a vector $\mathbf{x}^*$, the protection function β_i is defined as the following equivalent optimization problem:

$$\beta_i(\mathbf{x}^*, \Gamma_i) = \tag{2-14}$$

$$\text{Max} \quad \sum_{j \in J_i} \bar{a}_{ij} |x_j^*| z_{ij} \tag{2-15}$$

$$s.t. \quad \sum_{j \in J_i} z_{ij} \leq \Gamma_i \tag{2-16}$$

$$0 \leq z_{ij} \leq 1 \qquad \forall j \in J_i \tag{2-17}$$

To incorporate the proposed robustness criterion in a linear optimization model, a first bi-level (nonlinear) formulation is determined as follows:

$$\text{Max} \quad \mathbf{c^T x} \tag{2-18}$$

$$s.t. \quad \sum_j a_{ij} x_j + \beta_i(\mathbf{x}, \Gamma_i) \leq b_i \qquad \forall i \tag{2-19}$$

$$\mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \tag{2-20}$$

To reformulate as a linear optimization model, the authors have demonstrated that it is sufficient to replace the function $\beta_i(\mathbf{x}, \Gamma_i)$ by its dual equivalent. Next, the dual equivalent formulation for the β_i protection function is defined as follows:

$$\text{Min} \quad \sum_{j \in J_i} p_{ij} + \Gamma_i z_i \tag{2-21}$$

$$s.t. \quad z_i + p_{ij} \geq \bar{a}_{ij} |x_j^*| \qquad \forall i, j \in J_i \tag{2-22}$$

$$p_{ij} \geq 0 \qquad \forall j \in J_i \tag{2-23}$$

$$z_i \geq 0 \qquad \forall i \tag{2-24}$$

The final linear optimization formulation with additional variables and constraints is formulated as follows:

$$\text{Max} \quad \mathbf{c^T x} \tag{2-25}$$

$$s.t. \quad \sum_j a_{ij} x_j + \Gamma_i z_i + \sum_{j \in J_i} p_{ij} \leq b_i \qquad \forall i \tag{2-26}$$

$$z_i + p_{ij} \geq \bar{a}_{ij} y_j \qquad \forall i, j \in J_i \tag{2-27}$$

$$- y_j \leq x_j \leq y_j \qquad \forall j \tag{2-28}$$

$$l_j \leq x_j \leq u_j \qquad \forall j \tag{2-29}$$

$$p_{ij} \geq 0 \qquad \forall i, j \in J_i \tag{2-30}$$

$$y_j \geq 0 \qquad \forall j \tag{2-31}$$

$$z_i \geq 0 \qquad \forall i \tag{2-32}$$

In the development of the mentioned robust criticality criterion for the activities, subject of Chapter 4, we started with a non-linear formulation and then proposed a path-enumeration strategy to achieve a mixed-integer linear model where we could apply the Bertsimas and Sim's method.

In Chapter 5, the modeling of the portfolio problem demands the development of a new robust optimization approach. In this problem, we faced challenges that were not approached by the Bertsimas and Sim's method, despite having inspired our work. Correlated uncertainty factors affecting simultaneously rows and columns of the problem model and differentiated decision-dependent uncertainty budget requirements per risk are some of the features that motivated the development of this new robust optimization method. This method could be applied in different contexts that share the same characteristics. A research on its application to other problems is beyond the scope of the present research. However, in Chapter 7, this topic is proposed as a future research direction derived from this work.

3
Related Literature

In this chapter, we describe the literature on project management and control methods that are closely related with the research presented in this work. We divide this literature into the following three main fields: critical path analysis, multi-mode project scheduling, and project risk management. The first one explores methods to rank activities in terms of importance according with a pre-designed criterion. We call multi-mode project scheduling the field that examines problems related to devising effective resource allocation plans, given that activities could be executed under different conditions (or modes) and each one has their own resource requirements and associated durations. The latter concerns methods that study how to properly manage the risks associated with projects, indicating how to effectively represent, monitor, and mitigate them.

3.1
Critical Path Analysis

The community has early recognized the importance of using decision support methods for helping project management. In 1959, the development of the CPM (Kelley & Walker, 1959), and the PERT (Fazar, 1959) gave rise to this research field. The key idea of both techniques is the identification of the so-called project's critical path. The project is represented by a set of activities to be performed, with corresponding deterministic duration estimates and a set of precedence constraints (or precedence relations). The zero-lag finish-to-start relation is the most commonly used precedence constraint; it states that an activity a cannot start until its predecessor activity b has finished. The project's critical path is defined as the longest sequence of activities, connected by corresponding precedence relations that prevent the project from finishing earlier (Kelley, 1961). Any delay in critical activities also causes a delay in the project's duration. For this reason, project managers should pay close attention to the performance of these activities.

Figure 3.1 provides an example of the critical path of a project with five activities. We use a Gantt Chart representation of the project schedule, each

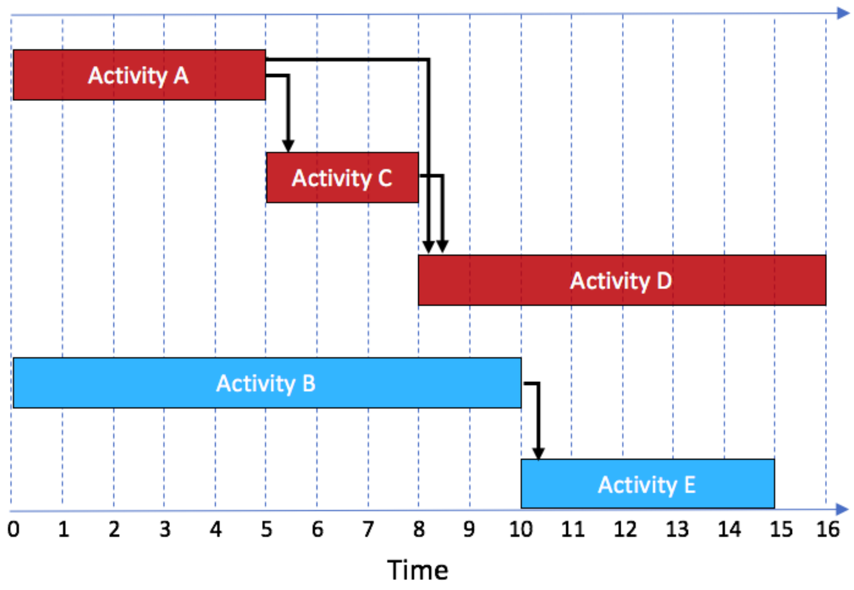


Figure 3.1: Gantt Chart with highlighted critical path.

activity is represented by a bar ranging from start to finish times, and each precedence is denoted by a predecessor to successor's arrow. In this example, activities are scheduled to their earliest possible starting times, which are constrained by finishing times of corresponding predecessors. The critical path is highlighted in red and is formed by the sequence of activities A, C, and D. We use this same project example in the next chapter.

An important limitation of the previously mentioned methods is the assumption that activity durations are known with certainty, and as a consequence, deterministic values are assumed for them. However, projects are subject to multiple sources of uncertainty that could directly impact their activities and potentially cause delays and cost overruns. Therefore, it is crucial to properly represent and deal with the uncertainty in the project management decision-making process. A common approach is to model the uncertainty in an activity-based fashion, where the durations of the activities are assumed to be uncertain and are represented by corresponding probability distributions. Within this context, a project will likely not have a single deterministic critical path, and the original concept of critical activities also needs to be revisited. This has encouraged researchers to create other activity criticality measures (Demeulemeester, 2002). Within this context, the path criticality index (PCI) and the activity criticality index (ACI) were the first criticality measures to project networks with activity durations modeled by probability distributions (Williams, 1992). The PCI is defined as the probability that a path p is of longest duration (i.e., the probability of p being a critical path) (Elmaghraby, 2000). The ACI concerns to the corresponding index for the activities, measuring the probability that each activity is part of a critical path (Williams, 1992). Other indexes, such as the significance index (see Williams (1992)) and the cruciality index (see Demeulemeester (2002)), attempt to capture the correlation between the uncertain activity durations and the total project duration. The calculation of these indexes is based on Monte-Carlo simulation methods, which require an enumeration of several uncertainty scenario realizations. Another form of representing uncertain parameters is by using fuzzy theory. Within this context, Chanas & Zieliński (2001) and Chen & Hsueh (2008) have devised criticality measures to project networks with fuzzy activity durations.

In Chapter 4, we propose a new criticality measure aimed at dealing with worst-case disruptions of the uncertainty. Based on robust optimization, this criterion also provides an upper bound for projects' durations under the adopted uncertainty hypothesis.

3.2
Multi-Mode Project Scheduling

The research on project control methods has naturally evolved from the critical path analysis to the study of more complex problems, modeling different decision-making realities. A key example is the class of problems that explore the time-cost trade-offs when activities could be executed in more efficient modes by extra allocation of resources. A well-studied problem of this class is the so-called discrete time-cost trade-off problem (DTCTP). In the DTCTP, each activity has a set of different modes that it can be executed, each one has a corresponding cost and duration, and the decision maker has to choose the best combination of modes to minimize a particular objective respecting a set of constraints. In the "deadline" variant of the DTCTP (DTCTP-d), the objective is to minimize the total cost to attend a pre-specified project deadline. In Akkan et al. (2005), they have proposed a decomposition technique based on column generation to solve the DTCTP-d. In Vanhoucke & Debels (2007), a set of different heuristic methods have been used to also solve the deadline problem with additional time-switch and work-continuity constraints. In DTCTP's "budget" version (DTCTP-b), the objective is to minimize the project's duration given a budget constraint that limits the total capacity investment on activity modes. In Öncü Hazır et al. (2010), they have proposed an algorithm based on a Benders decomposition to solve the DCTCP-b. A third version of the DTCTP is the so-called DTCTP-curve, the objective in this version is to build a complete efficient curve, which determines for each feasible project duration the minimum cost combination of activity modes that guarantees this duration. The study of Demeulemeester et al. (1998) has presented an iterative procedure that uses a branch-and-bound algorithm to determine every efficient solution of the DTCTP-curve. In addition, the works of Demeulemeester et al. (1996) and Hadjiconstantinou & Klerides (2010) have proposed flexible methods to solve all the three mentioned versions of the DTCTP (deadline, budget, and curve), with the first using a dynamic programming algorithm and the second developing a cut-generation solution based on path-enumeration. There are a number of papers that have also examined multi-mode trade-offs under the nomenclature of activity crashing problems (see, e.g., Rahimi & Seifi (2009); Doerner et al. (2008); Haga & O'keefe (2001)). Multi-mode problems are also studied under the variants of the resource constrained project scheduling problem (RCPSP), when the project has a limited number of resources to execute its activities and these activities have hard resource requirements (refer to Hartmann & Briskorn (2010) for a summary of works on the RCPSP and its variants). For an extensive list of papers on multi-mode problems, refer to Węglarz et al. (2011).

In the literature that explores time-cost trade-off problems, stochastic extensions to the DTCTP and its variants have also been proposed. For instance, in Klerides & Hadjiconstantinou (2010), activity durations have been assumed to be uncertain and have been represented by corresponding probability distributions, then they approached three two-stage stochastic extensions of the DTCTP: budget (SDTCTP-b), deadline (SDTCTP-d), and curve (SDTCTP-curve). In the first stage, the activity modes are decided, and in the second stage, after the uncertainty unfolds, the project scheduling is determined. The resulting two-stage problems are solved by a decomposition-based algorithm. In this stochastic literature, a number of papers have also used the approach of representing the time-cost trade-off on a single objective aimed at minimizing the project's total cost, which is assumed to be composed by activity costs and tardiness penalty costs (e.g., Gutjahr et al. (2000); Zhu et al. (2007); Tereso et al. (2004); Godinho & Branco (2012); Said & Haouari (2015)). Within this context, Gutjahr et al. (2000), Zhu et al. (2007), and Said & Haouari (2015) have investigated two-stage stochastic problems, while Tereso et al. (2004), Godinho & Branco (2012), and Klerides & Hadjiconstantinou (2015) have proposed adaptive policies for multi-stage decisions. In Chapter 5, we also represent the time-cost trade-off by a similar total cost minimization in the context of a portfolio of multiple interdependent projects.

As mentioned in Chapter 2, the robust optimization paradigm to decision under uncertainty is increasingly gaining attention, Gabrel et al. (2014). Its main idea is to provide solutions that guarantee a feasibility or objective criterion for all uncertainty scenarios comprised in the so-called uncertainty set. The robustness guarantees are achieved by a worst-case minimization or maximization under the uncertainty set. Most of the robust techniques provide the decision maker with a way to control the solution conservativeness by the adjustment of parameters that define the uncertainty set. In Öncü Hazır et al. (2011), the authors have proposed a robust variant for the DTCTP-d by using Bertsimas and Sim's approach (Bertsimas & Sim, 2004). As such, the uncertainty has been assumed to be in the activity costs, and the aim is to minimize the worst-case project's cost while guaranteeing that the deadline is attended. In Cohen et al. (2007), they have presented a multi-stage robust approach to the deadline version of the continuous time-cost trade-off problem (TCTP-d) with uncertainty regarding the activity durations. In Gutjahr (2015), each objective has been approached separately in a bi-objective method (time and cost), and they have used a strategy named optimization under multivariate stochastic dominance constraints (Dentcheva & Ruszczyński, 2009) to devise risk-averse solutions to an RCPSP multi-mode variant with uncertain durations.

This extensive multi-mode project scheduling literature highlights the importance of modeling different resource allocation strategies at the activity level. Therefore, we incorporate multi-mode activities in the methods developed in Chapters 5 and 6.

3.3 Project Risk Management

Project Risk Management (PRM) is the discipline that examines methods to identify, monitor, and control the effects of risks on a project (refer to the PMBOK® Guide, PMI, 2013). The identification is related with the processes of risk mapping, quantification, and prioritization. A risk is an uncertain event that, if materialized, affects multiple activities simultaneously, increasing their durations and costs. The set of mapped, quantified, and prioritized risks models the uncertainty environment of a project. An advantage of this approach, which is adopted in our method, is to allow for modeling correlated uncertainty impacts on activities (Hulett, 2009). In Creemers et al. (2014), the authors have also advocated, through extensive computational experiments, that representing the uncertainty by a risk-based approach is more effective than using the activity-based one, which is the common approach found in the previously detailed literature. The monitoring and control processes are related with assessing the effects of prioritized risks on project's objectives and providing effective response plans to minimize these effects. These risk management processes are a common and recommend practice for project managers, and could be implemented in different levels of data details and method sophistication. Extensive literature on these methods exists; they vary from qualitative analytics techniques (see Wang et al. (2004); Raz & Michael (2001); Raftery (2003)) to quantitative strategies (see Pate-cornell & Elisabeth (1996); Kılıç et al. (2008)). Some of these methods are well established in practice, and have been incorporated in the decision-making toolbox of project managers across different industries (e.g., Monte-Carlo Simulation, Hulett (2009)). In Muriana & Vizzini (2017) and Nguyen et al. (2013), authors have reviewed the PRM literature and reached the same conclusion that most of the methods focuses only on risk identification and assessment processes, and few methodologies provide decision support (DS) tools to prevent and control risk effects on project's planning and execution, which is the focus of this work. Important examples of methods in this class are RISKMAN (Carter et al., 1994), ARAMIS (Kirchsteiger et al., 1998), and PRAM (Chapman & Ward, 2003). Next, we review in more detail the DS systems that closely relate to our proposed methodology.

In Nguyen et al. (2013), authors have proposed a method called ProRisk. This method maps the set of risks and their corresponding impacts on activities and also the possible treatment (or mitigation) strategies to minimize these impacts. The method enumerates all possible combinations of risk scenarios and treatment strategies to predict the project performance under different conditions, which the authors have called project scenarios. As such, the decision-maker could analyze these project scenarios to determine the effective treatment plan to be adopted. Yet, the number of possible project scenarios grows exponentially with the number of risks and treatments strategies, which limits the application of this methodology. In Gładysz et al. (2015), the model maps a cost associated with the complete elimination of each risk. The authors have proposed a mixed-integer linear programming model to determine the minimum cost combination of risks selected for elimination to guarantee that the project's longest path, in terms of duration expected value, is less than a pre-defined deadline. The impossibility of complete risk elimination in some cases and ignoring possible penalties costs due to delays present some of the limitations of this methodology. In Kılıç et al. (2008), a genetic algorithm has been proposed to solve the bi-objective problem of minimizing simultaneously the expected values of project cost and longest duration path. The project cost is composed of the following four factors: overhead cost, activity execution cost, tardiness penalties, and the cost of implementing the selected risk-reduction strategies. These methodologies have been developed in the context of a single project.

As mentioned previously, given the increasing complexity of organizations, several authors (see Kearney (2016); Hans et al. (2007); Öncü Hazır (2015)) have advocated for the necessity of developing DS methods to manage portfolios of projects. For instance, Hans et al. (2007) have reviewed several studies in hierarchical planning and have proposed a generic framework to combine different existing techniques into a methodology for multi-project planning and control under uncertainty. The combinations of techniques are dependent on the projects' complexity and interdependency. The methods developed in the present work could be used as components applied by these hierarchical frameworks.

The new methodology developed in this work combines elements of the three described areas. The objective of this combination is to investigate the potential of the robust optimization paradigm applied in the current project (or portfolio) risk management practice.

4 Robust Activity Criticality Criterion and Uncertainty Mitigation for Project Duration

In this chapter, we present the first application of the proposed robust approach that models the uncertainty environment as an adversary that selects the worst-case scenario for any decision maker's actions. Given an activity-based model for the uncertainty faced by a project, the proposed approach attempts to capture the importance of the activities in a combined fashion using a robust optimization framework.

The modeled problem aims at determining the activities of a project that should be the focus of uncertainty mitigation measures – in the sense that resource and effort should be put into place so as to ensure that their actual durations equal their original nominal estimates – to control the project's execution. As described in Chapter 3, when considering uncertain activity durations, the criticality indexes found in the literature are mostly obtained through an extensive enumeration of scenarios under a Monte-Carlo simulation framework. Instead of doing so, we adopted a robust optimization approach that prioritizes activities to minimize the worst-case project's duration over the scenarios comprised in a controlled uncertainty set. As detailed ahead, this approach avoids the referred extensive enumeration of the uncertainty scenarios and provides a project's duration guarantee over the uncertainty set scenarios.

4.1 Problem Modeling

A project is composed of a set of activities that have to performed in order to complete it. The execution of these activities is subject to multiple sources of uncertainty, for instance, the productivity of the resources, problems with required machinery or tools, and weather uncertainty. A common approach to model this uncertainty is to provide optimistic, nominal, and pessimistic estimates for the duration of each activity. These estimates are then used as input to define probability distributions for the corresponding durations (Hulett, 2009). Due to the difficulty and lack of information, these distributions are assumed to be independent, meaning that the correlation between them is ignored. As pointed out in Chapter 3, in this context, the critical path of the project is also uncertain and depends on the realization of the activity durations. In this work, we propose a new criticality criterion that is based on robust optimization and that could be determined by a cut-generation solution algorithm. In the following sections, we detail the representation used for the activity network and the corresponding problem statement.

4.1.1 Problem Statement

Given a project composed of a set of activities A and precedence relations E, we used an Activity-on-Node AoN network representation $G = (A, E)$. In this network, nodes represent the activities and edges the precedence relations; the next section provides an example of this network representation. For each activity a, a nominal duration d_a and a potential increase in this duration due to the uncertainty Δ_a are defined. The value $d_a + \Delta_a$ represents the worst-case estimate for the duration of a. Each precedence relation is denoted by predecessor and successor activities i and j respectively, which are represented here as (i, j). A precedence (i, j) constrains j to only start its execution once i is finished (i.e., finish-to-start zero-lag relation).

Our model attempts to answer the following basic question: given a project represented by $G = (A, E)$, what are the α most important activities that one should guarantee to be performed within their nominal duration estimates (for example, by allocating additional resources or implementing uncertainty-mitigation measures) to minimize the project's duration T, under the assumption that, at most, β activities will assume their worst-case durations? As detailed ahead, the β parameter relates to the pessimism about the overall project's execution performance, while worst-case duration estimates denote a local performance assumption at the activity level. Under such hypothesis, the model provides the optimal set of activities that should be the target of uncertainty mitigation and an upper bound on the project's total duration T – which is arguably useful for project managers.

4.1.2 Example

We start by illustrating the AoN network representation and its corresponding project's duration (T) calculation. In Table 4.1, we define the instance parameters for each activity that composes our project example. In addition, Figure 4.1 presents a representation of the AoN network in the case where α

Activity, a	d_a	Δ_a	*Predecessors*	*Successors*
A	5	2	-	C, D
B	10	6	-	E
C	3	3	A	D
D	8	2	A, C	-
E	5	3	B	-

Table 4.1: Robust criticality example's instance data.

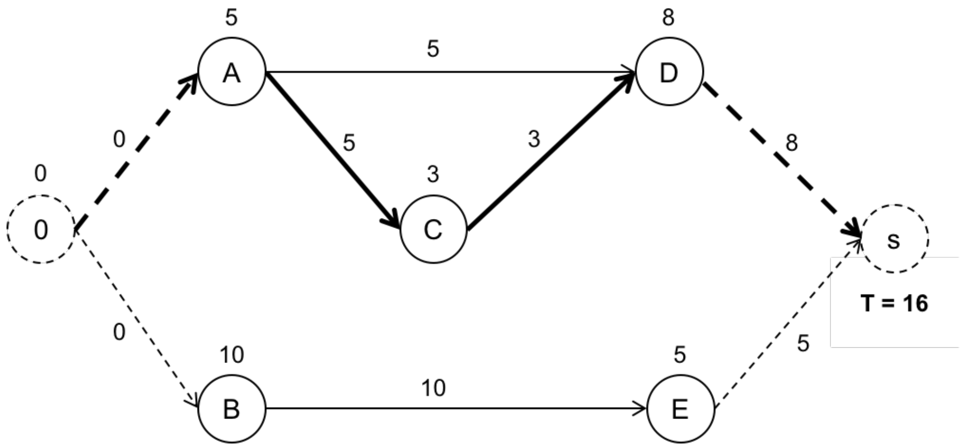


Figure 4.1: *AoN* network of Table's 4.1 instance with $\alpha = 0$ and $\beta = 0$.

and β are zero, which entails that no activity is target of risk-mitigation measures or of uncertainty impacts. The activity durations are then represented on top of their corresponding nodes, and each precedence (i, j) is assigned its predecessor's duration d_i as its corresponding weight, following the representation proposed in Bartusch et al. (1988). The main objective of this representation is to be able to apply standard network-flow models and algorithms to determine activities' early schedules and the resulting project duration (T). To achieve this objective, dummy (zero duration) source and sink activity nodes had to be added to the network. The source activity, represented by node 0, is connected by precedence relations to all activities that do not have predecessors, allowing all nodes to be reachable from this source node. The sink dummy activity, represented by node s, is added to the network and has all activities that do not have successors as predecessors. With this representation, the early-start schedule of each activity a, is simply the weight of the longest path from source node 0 to a, and the project duration T is given by the early-start of the sink node s. In Figure 4.1, we highlight in bold the project's critical path, which is the longest path from source 0 to sink s. In this network, the critical path is composed by activities A, C, and D, resulting in total duration

$T = 16$. Given the fact that circular dependencies are not allowed, the network G is a directed acyclic graph (DAG); hence, the problem of determining the longest path from source to any other node admits a polynomial-time solution algorithm (Cormen et al., 2001). This problem also has a network-flow integer programming formulation with a totally unimodular constraint matrix, which, consequently, has the same optimal solution of its corresponding linear relaxation (Papadimitriou & Steiglitz, 1982).

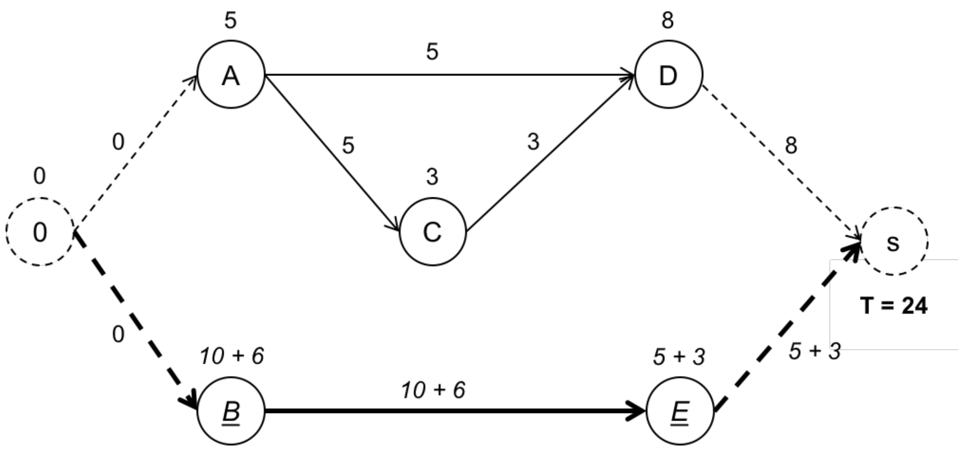


Figure 4.2: *AoN* network of Table's 4.1 instance with $\alpha = 0$ and $\beta = 2$.

In Figure 4.2, we illustrate the same instance of Table 4.1 but with the adversary having the option to set two activities to their worst-case duration estimates (i.e., $\alpha = 0$ and $\beta = 2$). The adversary decision that maximizes the project's duration is to impact activities B and E. This combination delays the project to $T = 24$, as B is impacted with $\Delta_B = 6$ and E with $\Delta_E = 3$. These impacts are represented in Figure 4.2 by summing nominal durations to corresponding Δ values. The representation also highlights the change in the critical path, which is then formed by activities B and E.

The final network, presented in Figure 4.3, represents the case where $\alpha = 2$ and $\beta = 2$, that is, the decision maker now has the option to avoid uncertainty impacts on two activities. The optimal decision would be to mitigate the uncertainty for B and C, which is illustrated by filling their corresponding nodes with gray. In this case, the worst-case scenario that could be materialized by the adversary would be to impact activities A and D, which results in a worst-case project duration of $T = 20$. By selecting activities B and C, the decision maker is simultaneously protecting the two potential critical paths, instead of just focusing the original one presented in Figure 4.1. Next, we provide the corresponding mathematical models and solution strategy for the proposed problem.

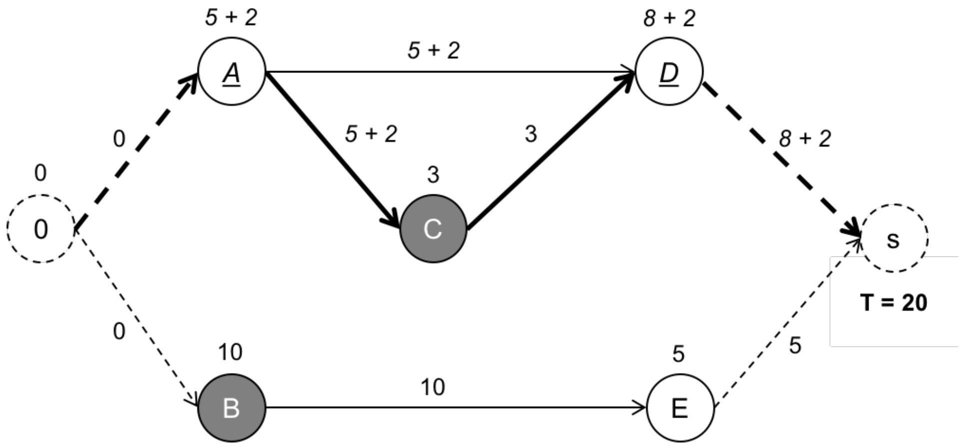


Figure 4.3: AoN network of Table's 4.1 instance with $\alpha = 2$ and $\beta = 2$.

4.2 Mathematical Formulations and Cut-Generation Algorithm

4.2.1 Min-Max Model

We start by presenting a bi-level (nonlinear) optimization model which is then reformulated into a robust linear model. The first level represents the decisions regarding which α activities (x) should be target of risk-mitigation measures. While the second level represents the adversary's action, that is, selection of the β activities (z) that were not target of risk-mitigation measures in first-level decisions, which assuming their worst-case durations, will maximize the project's total duration T. Subsequently, we provide parameters and decision variables definitions followed by first and second-level models:

d_a	nominal duration of activity a;
Δ_a	potential increase in duration of activity a associated with the uncertainty;
α	maximum number of activities for which the uncertainty could be mitigated;
x_a	binary decision variable that indicates if an uncertainty mitigation measure associated with activity a was implemented, which in the positive case ($x_a = 1$), guarantees a duration equals to d_a;
β	maximum number of activities that could assume worst-case durations (i.e., $d_a + \Delta_a$);
z_a	binary decision variable that indicates if activity a will assume its worst-case duration $d_a + \Delta_a$;

$\Pi(x)$	adversary (or second-level) function that for a given first-level decision x, returns the maximum project duration that could be achieved when at most β non-mitigated activities (i.e., $x_a = 0, a \in A$) assume their worst-case durations;
δ_a^-	set of predecessors of activity a ($\delta_a^- = \{i \mid (i,a) \in E\}$);
δ_a^+	set of successors of activity a ($\delta_a^+ = \{j \mid (a,j) \in E\}$);
u_{ij}	auxiliary network-flow variable that indicates if the precedence arc (i,j) is part of the longest-path from source node (0) to sink node (s).

$$\underset{x}{\text{Min}} \quad \Pi(x) \tag{4-1}$$

$$s.t.$$

$$\sum_{a \in A} x_a \leq \alpha \tag{4-2}$$

$$x_a \in \{0,1\} \qquad \forall a \in A \tag{4-3}$$

where:

$$\Pi(x) = \underset{z,u}{\text{Max}} \sum_{(i,j) \in E} (d_i \cdot u_{ij} + (1 - x_i) \cdot \Delta_i \cdot z_i) \tag{4-4}$$

$$s.t.$$

$$\sum_{j \in \delta_0^+} u_{0j} = 1 \tag{4-5}$$

$$\sum_{i \in \delta_s^-} u_{is} = 1 \tag{4-6}$$

$$\sum_{i \in \delta_a^-} u_{ia} - \sum_{j \in \delta_a^+} u_{aj} = 0 \qquad \forall a \in A,\ a \notin \{0, s\} \tag{4-7}$$

$$\sum_{a \in A} z_a \leq \beta \tag{4-8}$$

$$z_a \leq \sum_{j \in \delta_a^+} u_{aj} \qquad \forall a \in A \tag{4-9}$$

$$u_{ij} \in \{0,1\} \qquad \forall (i,j) \in E \tag{4-10}$$

$$z_a \in \{0,1\} \qquad \forall a \in A \tag{4-11}$$

The objective function of the model 4-1 is simply to minimize, over x decisions, the project's total duration due to the adversary action, which is represented by the function $\Pi(x)$. Constraint 4-2 limits the number of activities that are subject of uncertainty mitigation measures.

In the adversary's equivalent optimization model $\Pi(x)$, we applied a network-flow formulation, using the AoN representation to capture the longest path from source to sink. Constraints 4-5, 4-7, and 4-6 guarantee, respectively, that the network flow path (u) starts at the source node 0, flows through intermediate nodes (or activities), and ends at the sink node s. Constraints 4-9 guarantee that only activities part of the longest path from source to sink could assume their worst-case durations, while constraint 4-8 limits the total number of these activities. The first term of the adversary's objective function 4-4 reflects the size of the referred longest path, while the second one maps the potential impact due to the uncertainty for the selected activities (i.e., $z_a = 1, a \in A$).

However, techniques that allows us to solve this bi-level (non-linear) model in this current form are not readily available. To circumvent such difficulty, we propose a reformulation based on the enumeration of all paths from source 0 to sink s. This reformulation allows for the development of a one-level linear model. In the subsequent section, we detail this reformulation scheme.

4.2.2 Path Enumeration Model

Let q be a sequence of activities that forms a path from source 0 to sink node s, Q the set of all possible paths, and t_q the duration of path q, the model can be reformulated as follows:

$$\underset{x,T,t,z}{\text{Min}} \quad T \tag{4-12}$$

$$s.t.$$

$$T \geq t_q \qquad \forall q \in Q \tag{4-13}$$

$$t_q = \underset{\substack{z \\ \sum z_a \leq \beta}}{\text{Max}} \sum_{a \in q} (d_a + (1 - x_a) \cdot \Delta_a \cdot z_a) \qquad \forall q \in Q \tag{4-14}$$

$$\sum_{a \in A} x_a \leq \alpha \tag{4-15}$$

$$x_a, z_a \in \{0, 1\} \qquad \forall a \in A \tag{4-16}$$

Constraints 4-14 guarantee that t_q is set to the worst-case duration of q, given the uncertainty mitigation decisions x and allowing, at most, β activities to assume their worst-case durations (i.e., $d_a + \Delta_a$). The maximization problem in the right-hand side (RHS) of constraints 4-14 models the adversary's action for a particular path q. Constraints 4-13 guarantee that T is set to the maximum duration over all paths $q \in Q$.

Although the model cannot yet be solved in a straightforward manner, it now allows for the application of the techniques presented in (Bertsimas & Sim, 2004): the maximization problem on the RHS of each of the constraints defined in (4-14) may be substituted by the objective function of its dual, while the dual feasibility constraints are incorporated into the outer minimization problem. Once this transformation is carried out, we are left with a mixed-integer linear programming problem that can be solved by commercially available solvers. Next, we detail this transformation.

The adversary's problem for a particular path q is represented by the following function:

$$\Pi_q(x) =$$

$$\max_{z} \sum_{a \in q} (d_a + (1 - x_a) \cdot \Delta_a \cdot z_a) \tag{4-17}$$

$$s.t.$$

$$\sum_{a \in q} z_a \leq \beta \tag{4-18}$$

$$z_a \leq 1 \qquad \forall a \in q \tag{4-19}$$

$$z_a \geq 0 \qquad \forall a \in q \tag{4-20}$$

We use the linearly relaxed version, that is, constraints 4-19 and 4-20 instead of $z_a \in \{0, 1\}, \forall a \in q$, due to the equivalence of both versions. According to Papadimitriou & Steiglitz (1982), a linear optimization model with constraints in the form $Ax \leq b, x \geq 0$, having b with integer elements and A being totally unimodular (TUM), is equivalent to its integer version (i.e., the case where elements of x are also required to be integers). The constraint matrix of $\Pi_q(x)$ is TUM because of the following theorem presented in Papadimitriou & Steiglitz (1982):

Theorem 1 *An integer matrix A with $a_{ij} = 0, +1, -1$ is TUM if no more than two nonzero entries appear in any column, and if the rows of A can be partitioned into two sets I_1 and I_2 such that:*

1. *If a column has two entries of the same sign, their rows are in different sets;*

2. *If a column has two entries of different signs, their rows are in the same set.*

The application of Theorem 1 to the constraint matrix of $\Pi_q(x)$ is straightforward, since the set of constraints 4-18 and 4-19 could be used as partitions I_1

and I_2. As $\Pi_q(x)$ is a maximization problem, we could also ignore constraints 4-20.

The dual equivalent of the previous model is defined as follows:

π_q	dual decision variable of path q due to constraint 4-18;
γ_{aq}	dual decision variable of path q, activity a due to constraints 4-19.

$$\Pi_q(x) = \min_{z} \sum_{a \in q} d_a + \beta \cdot \pi_q + \sum_{a \in q} \gamma_{aq} \tag{4-21}$$

$s.t.$

$$\gamma_{aq} + \pi_q + \Delta_a \cdot x_a \geq \Delta_a \qquad \forall a \in q \tag{4-22}$$

$$\gamma_{aq} \geq 0 \qquad \forall a \in q \tag{4-23}$$

$$\pi_q \geq 0 \tag{4-24}$$

As previously mentioned, we can replace the maximization problem on the RHS of constraints 4-14 to its dual equivalent formulation to achieve a one-level mixed-integer programming (MIP) model:

$$\min_{x,T,t,\pi,\gamma} \quad T \tag{4-25}$$

$s.t.$

$$T \geq t_q \qquad \forall q \in Q \tag{4-26}$$

$$t_q = \sum_{a \in q} d_a + \beta \cdot \pi_q + \sum_{a \in p} \gamma_{aq} \qquad \forall q \in Q \tag{4-27}$$

$$\gamma_{aq} + \pi_q + \Delta_a \cdot x_a \geq \Delta_a \qquad \forall q \in Q,\ \forall a \in q \tag{4-28}$$

$$\sum_{a \in A} x_a \leq \alpha \tag{4-29}$$

$$x \in \{0, 1\} \qquad \forall a \in A \tag{4-30}$$

$$\gamma_{aq} \geq 0 \qquad \forall q \in Q, \forall a \in q \tag{4-31}$$

$$\pi_q \geq 0 \qquad \forall q \in Q \tag{4-32}$$

This model could be solved directly by commercially available solvers. Nonetheless, a difficulty that arises is the fact that the number of paths in a DAG ($|Q|$) could grow exponentially with its number of nodes. Without loss of generality, consider a DAG $G^* = (A, E)$ with node indexes labeled in topological order, where, for all pair of nodes $(i, j), i < j$, there is an edge connecting them (i.e., $(i, j) \in E$). For each subset of A, we could construct a path sequence by simply sorting its elements according with their node indexes.

Then, the total number of distinct paths in G^* is $2^{|A|}$, which is the number of subsets in A. To overcome this difficulty, we propose a cut-generation algorithm that is presented in the next section.

4.2.3 Cut-Generation Algorithm

The idea of the algorithm is to maintain only a subset of paths ($SQ \subseteq Q$) in the master problem. A separation procedure is called at each iteration to identify whether a path, which is not in SQ, should be considered. In this case, the path is included in the master problem and the algorithm proceeds. Otherwise, the set of activities that should be target of uncertainty mitigation measures x obtained by solving the master problem is optimal. The separation problem is exactly the optimization problem equivalent to the function $\Pi(x)$ (see Section 4.2.1). The solution of this problem will return the longest path that could be achieved by setting at most β non-mitigated activities (i.e., $a|x_a = 0$) to their worst-case durations. The path itself is recovered from the values of network-flow variables u_{ij} and the value of the objective function is its corresponding duration. Algorithm 1 outlines the cut-generation solution strategy in greater detail.

Given the first-level decisions x, the separation problem $\Pi(x)$ turns into an MIP model that could be solved by commercially available solvers. In what follows, we illustrate that this problem could also be solved directly by a polynomial-time solution algorithm.

Solving Separation in Polynomial-Time

We start by defining a recurrence function $ES(j, x, b)$, equations 4-33 and 4-34, which is equivalent to the worst-case (longest) early-start schedule of activity j, given first-level decisions x and with at most b activities assuming worst-case durations ($d_j + \Delta_j$). With this definition, the separation (adversary) function $\Pi(x)$ is equivalent to $ES(s, x, \beta)$, that is, the worst-case early start of sink node s with at most β activities assuming worst-case durations.

$$ES(0, x, b) = 0 \tag{4-33}$$

$$ES(j, x, b) = \mathop{\mathbf{Max}}_{i \in \delta_j^-} \left\{ \begin{array}{c} ES(i, x, b) + d_i; \\ ES(i, x, b-1) + d_i + \Delta_i, \ \textbf{if} \ x_i = 0 \ \textbf{and} \ b \geq 1. \end{array} \right. \tag{4-34}$$

Equation 4-33 defines the recurrence base case, for which the source node (0) early-start is set to zero for any x and b. Equation 4-34 describes the

Algorithm 1 Robust Critical Activities Cut-Generation

$SQ \leftarrow \emptyset$ ▷ SQ initialization
$q \leftarrow [0, s]$ ▷ Dummy path
$LB \leftarrow 0$ ▷ Lower Bound initialization
$UB \leftarrow \infty$ ▷ Upper Bound initialization
$BestSol \leftarrow \emptyset$ ▷ Optimal Solution initialization
$MP \leftarrow$ Create master problem from the final model of section 4.2.2
while $LB < UB$ **and** $ElapsedTime < TimeLimit$ **do**
    Add q to SQ and its corresponding cut to MP (variables and constraints related to q from the final model of section 4.2.2)
    $MPSol \leftarrow$ Solve MP to obtain its current solution
    $LB \leftarrow MPSol.obj$ ▷ LB is updated with the objective value of $MPSol$
    if $LB = UB$ **then**
        break
    end if
    $x \leftarrow MPSol.x$ ▷ Get x decisions from $MPSol$
    $SP \leftarrow$ Create separation problem for current x according with the model of $\Pi(x)$
    $SPSol \leftarrow$ Solve SP to obtain its current solution
    $q \leftarrow$ Recover path from $SPSol.u$
    $T \leftarrow SPSol.obj$
    if $T < UB$ **then**
        $BestSol \leftarrow x$
        $UB \leftarrow T$
    end if
end while
return $BestSol$

general case, which is based on the fact that in a project network with finish-to-start precedences, the early-start of an activity j is defined by the maximum early-finish between their predecessors $i \in \delta_j^-$. The RHS of the brackets define possibly two cases for the worst-case early-finish of a particular predecessor i. The first one maps the case where i assumes its nominal duration d_i, which does not require a decrease in b, so the considered early-start of i is simply $ES(i, x, b)$. The second one represents the case where i assumes its worst-case duration $d_i + \Delta_i$. This case is only considered when i is not target of risk mitigation measures (i.e., $x_i = 0$) and the maximum number of worst-case duration activities was not reached (i.e., $b \geq 1$). The fact that i is assuming its worst-case duration requires b to be decreased; hence, the early-start of i in this case should consider this requirement, which is then mapped to $ES(i, x, b-1)$. Next, we present a dynamic programming polynomial-time algorithm to solve the referred recurrence.

Given the network $G(A, E)$ with $n = |A|$ and $m = |E|$, and without

Algorithm 2 Robust Critical Activities Polynomial-Time Separation

```
var ES[n+2][β]
var Pred[n+2][β]
var PredImp[n+2][β]
for b = 0 to β do:
    for j = 0 to n+1 do:
        ES[j][b] ← 0
        Pred[j][b] ← 0
        PredImp[j][b] ← False
    end for
end for
for b = 0 to β do:
    for j = 1 to n+1 do:
        for i ∈ δ_j^- do:
            EF_iNoImp ← ES[i][b] + d_i
            if EF_iNoImp > ES[j][b] then:
                ES[j][b] ← EF_iNoImp
                Pred[j][b] ← i
                PredImp[j][b] ← False
            end if
            if x_i = 0 and b ≥ 1 then:
                EF_iImp ← ES[i][b−1] + d_i + Δ_i
                if EF_iImp > ES[j][b] then:
                    ES[j][b] ← EF_iImp
                    Pred[j][b] ← i
                    PredImp[j][b] ← True
                end if
            end if
        end for
    end for
end for
T ← ES[n+1][β]
q ← ∅
z ← ∅
CurrAct ← n+1
CurrB ← β
while Pred[CurrAct][CurrB] <> 0 do:
    CurrPredImp ← PredImp[CurrAct][CurrB]
    CurrAct ← Pred[CurrAct][CurrB]
    q ← CurrAct ∪ q
    if CurrPredImp = True then:
        z ← CurrAct ∪ z
        CurrB ← CurrB − 1
    end if
end while
return T, q, z
```

loss of generality, Algorithm 2 assumes that the activity nodes are indexed in topological order; as such, if $(i,j) \in E$ then $j > i$, with source and sink nodes indexed by 0 and $n+1$ respectively. The algorithm works by fulfilling three $n+2 \times \beta$ matrices: ES, $Pred$ and $PredImp$. The element $ES[j][b]$ stores the actual recurrence value $ES(j,x,b)$, while $Pred[j][b]$ stores which predecessor of j constrained its early start and $PredImp[j][b]$ indicates whether the corresponding predecessor was impacted by the uncertainty and then assumed its worst-case duration. From lines 4 to 10, the refereed matrices are initialized to implement the recurrence base case. From lines 11 to 30, the algorithm implements the general case by iterating over increasing values of b, from 0 to β, and, in topological order of activities, from 1 to $n+1$. This iteration order is essential to guarantee that when a predecessor recurrence value is used (i.e., $ES[i][b]$ and $ES[i][b-1]$ at lines 14 and 21 respectively), it has already been correctly determined. From lines 13 to 28, the algorithm iterates over the predecessors of j and analyzes the recurrence cases of equation 4-34, the first one being implemented from lines 14 to 19 and the second one from lines 20 to 27. The final part of the algorithm, lines 31 to 45, implements the retrieval of the recurrence solution itself, recovering the longest-path q, its total duration T and the set of activities that assumed worst-case durations z.

Given that $\beta \leq n$, the initialization complexity is $O(n^2)$. The loop from lines 12 to 29 takes, on aggregate, $O(m)$ operations. Therefore, the main loop complexity is $O(n*m)$, lines 11 to 30. The solution's retrieval part, lines 31 to 45, only takes $O(n)$ operations. Due to the edges that are included to connect source and sink nodes (see 4.1.2), m is $\Omega(n)$, and consequently the algorithm's complexity is dominated by the main loop, which is $O(n*m)$.

4.3 Computational Experiments

The main idea of the designed experiments was to assess the effectiveness of traditional critical path analysis in the context of a combined worst-case uncertainty effect on project activities. The proposed robust criterion was designed to capture this worst-case uncertainty scenario.

We performed our experiments on project networks extracted from the 480 *PSPLIB* instances for the RCPSP with $n = 30$ jobs (or activities) (Kolisch & Sprecher, 1997). From this set of instances, we used the information about the activity durations d_a and precedence relationships. To assign worst-case duration estimates, for each activity a, we first defined a Beta-PERT distribution with parameters $0.8 * d_a$, d_a and $1.4 * d_a$ as optimistic, most likely, and pessimistic estimates (see Slyke & Richard (1963)). Then, we set the 0.95

quantiles of corresponding distributions as worst-case durations ($d_a + \Delta_a$).

For every instance, we solved our model for each combination of α and β where $\alpha \geq 0$, $\beta \geq 1$ and $\alpha + \beta \leq 30$. Tests were performed on an Intel Core i5-3360M PC with 4 cores of 2.80GHz and 8 GB of RAM. The model was implemented using the programming language Python and solved by IBM(R) ILOG(R) CPLEX(R) 12.5.0.0.

In Figure 4.4, we analyzed how the increase on the number of mitigated activities α affects the delay relative to the original project duration, given a fixed assumption on the number of activities that take on their worst-case duration – in particular, the chart displays results for each combination of $(\beta, \alpha) \in \{(10, 2), (10, 4), (10, 6), (10, 8)\}$. Given that T represents the original total project duration and T_r denotes the robust estimate of total project duration (i.e., the total duration of the project as per the result of the optimization model), the y-axis represents the delay expressed in percentage points ($D = 100 * \frac{(T_r - T)}{T}$) for each of the instances represented on the x-axis (sorted in ascending order of delay). Moreover, a significant decrease on delays as α increases can be observed, which suggests how imperative it is to correctly select the set of activities to have theirs risks mitigated.

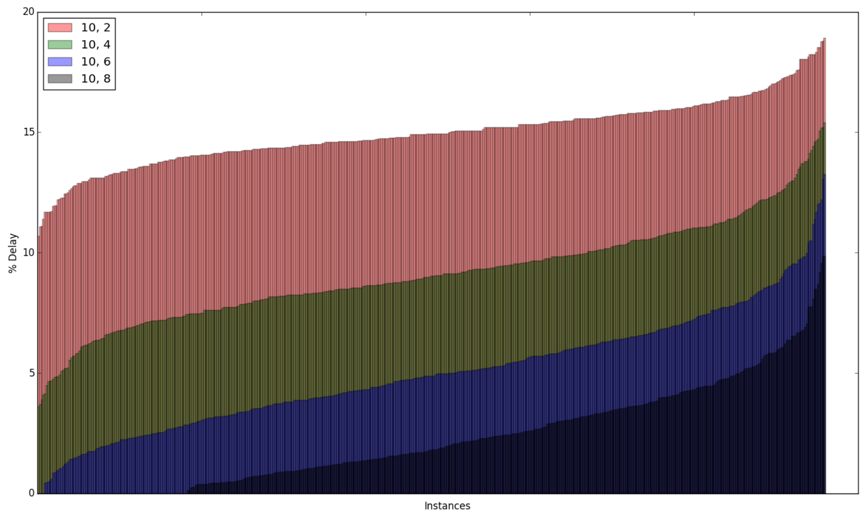


Figure 4.4: Ordered total percent delay by instance for $\beta = 10$ and $\alpha = \{2, 4, 6, 8\}$.

Another key fact, evident in Figure 4.4, is that for the instances which present higher values of D, increasing α has a progressively lower impact on the actual decrease of the delay. This is evidence that in complex project networks, it becomes increasingly difficult to mitigate the impact of simultaneous delays in multiple activities. To analyze this fact and the quality of our approach, we selected the following two representative instances: *j3028_ 10.sm* and *j3025_ - 9.sm*. The first instance (*j3028_ 10.sm*) represents the instances in which it is easier to determine the best combination of activities to mitigate, that is, the instances where a traditional critical path analysis could successfully be used to perform this task. On the other hand, instance *j3025_ 9.sm* represents the case of complex networks where the traditional critical path analysis would be expected to perform poorly.

Instance	β	α	T	T_r	D	*Mitigated Activities*
j3028_10.sm	10	2	59	68.12	15.45%	[**19**, **26**]
j3028_10.sm	10	4	59	64.69	9.64%	[**7**, **19**, **26**, **28**]
j3028_10.sm	10	6	59	61.50	4.23%	[**7**, **9**, **19**, **23**, **26**, **28**]
j3028_10.sm	10	8	59	59.45	0.76%	[**1**, **7**, **9**, **19**, **22**, **23**, **26**, **28**]
j3025_9.sm	10	2	50	59.11	18.22%	[3, **26**]
j3025_9.sm	10	4	50	57.20	14.40%	[2, 3, **5**, **26**]
j3025_9.sm	10	6	50	56.52	13.04%	[3, **5**, 8, **10**, 24, **26**]
j3025_9.sm	10	8	50	54.92	9.84%	[3, **5**, 8, **10**, 13, 16, 24, **26**]

Table 4.2: Results for instances j3028_10.sm and j3025_9.sm with $\beta = 10$ and $\alpha = \{2, 4, 6, 8\}$.

Table 4.2 details the results of the above mentioned selected instances. In the final column, *Mitigated Activities*, we provide the set of activities (or jobs), which were selected as part of the optimal solution obtained by our model for the corresponding instance and values of β and α. The activities in bold are those that are part of a critical path (*i.e.*, the activities that would traditionally be defined as critical). We first noticed that, as α increases, decreases in delay for *j3028_ 10.sm* are higher than those for *j3025_ 9.sm*. Second, all the activities on the solutions for *j3028_ 10.sm* are critical activities, while for *j3025_ 9.sm* less than half of them are. These results indicate that in complex cases, the traditional critical path analysis cannot adequately capture the combined effects of worst-case durations, highlighting the main advantage of our approach.

We subsequently analyzed all the results obtained for each tested combination of α and β for both instances. To accomplish this, we built three different charts to summarize the results of all combinations for which we solved the model. Figures 4.5 and 4.6 present the charts for *j3028_ 10.sm* and *j3025_ 9.sm*, respectively:

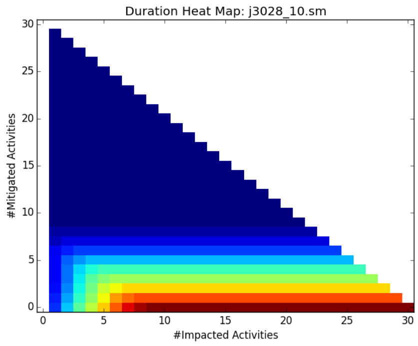
1. The **duration heat map** chart captures the duration result of our model for each tested combination of α and β. The x-axis represents the number of impacted activities (β), while the y-axis denotes the number of mitigated activities (α). Colors are assigned based on the corresponding duration in each point (combination of α and β) following a heat-scale ranging from dark-blue (no delay) to dark-red (maximum delay).

2. The **mitigation frequency** chart displays the number of times that each activity (job) is on the optimal solution of the tested combinations, that is, the frequency that the activity was selected to be mitigated. On the x-axis, we have the activity numbers ordered by frequency, while on the y-axis, we have the corresponding frequencies. Red bars indicate critical activities, whereas blue bars are used for the non-critical ones. This is somewhat analogous to the evaluation performed in the calculation of an activity's criticality index but aims to provide evidence as to how the traditional analysis could be expected to perform.

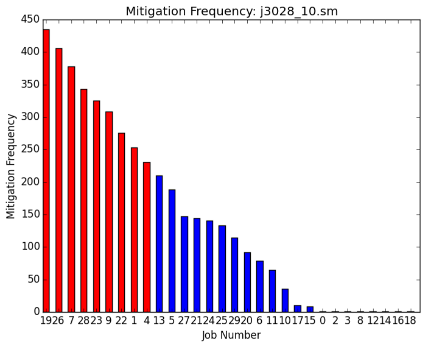
3. **Gantt chart** illustrates the original schedule of the activities, respecting the precedence relations and with their durations represented by the bar sizes. The critical activities are highlighted with borders in bold. The redness of each activity is assigned proportionally to its mitigation frequency.

The duration heat map of *j3028_10.sm* (Figure 4.5(a)) demonstrates that, on a large portion of the tested combinations, the mitigation was capable of avoiding any delay. In fact, for all combinations with $\alpha \geq 9$, it was possible to maintain the original project duration. As we could expect, 9 is the exact number of critical activities. Another important fact is revealed by the mitigation frequency graphic 4.5(b): all critical activities have a higher mitigation frequency than any non-critical activity.

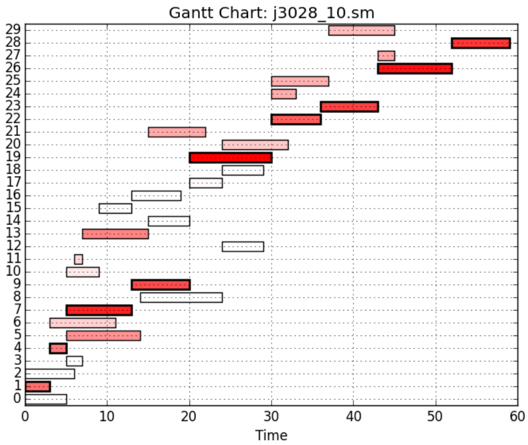
By examining the graphs for instance *j3025_9.sm* in Figure 4.6, the same behavior that was exhibited in Table 4.1 for this particular instance can be observed. The color variation, demonstrated in its duration heat map (Figure 4.6(a)) as α increases, is smoother than the one for the *j3028_10.sm*, which entails that the impact of increasing the number of mitigated activities in *j3025_9.sm* is not as effective as in *j3028_10.sm*. In fact, in *j3025_9.sm*, only for $\alpha \geq 18$ it is possible to guarantee that the project will not be delayed – which is surprising since that there are only 7 critical activities. The mitigation frequency graphic (Figure 4.6(b)) also provides evidence of the complexity of this particular instance; many of the non-critical activities have mitigation frequencies that are comparable – and often higher – than those of critical



4.5(a): Duration Heat Map



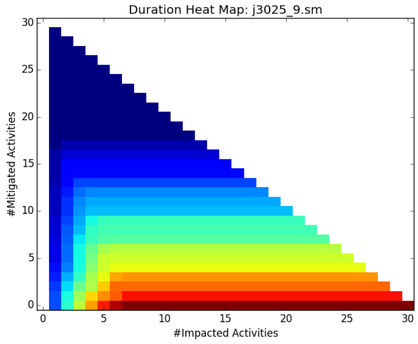
4.5(b): Mitigation Frequency



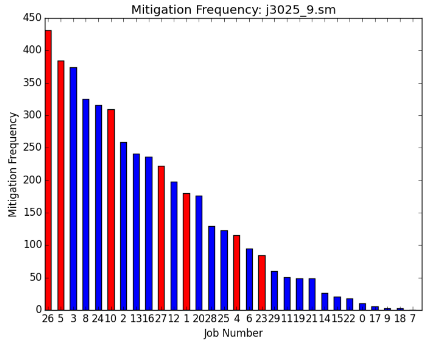
4.5(c): Gantt Chart

Figure 4.5: Result graphics for the j3028_10.sm instance.

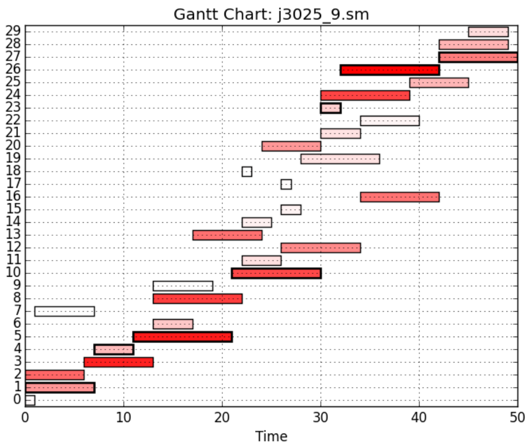
ones. In this case, focusing mitigation measures to avoid delays exclusively on critical activities would not be an effective strategy to ensure shortest project duration. Finally, it is noteworthy that the comparison of the two project networks offers no evidence as to which project is more complex, which again suggests that our proposed approach might prove useful across projects with diverse characteristics. In fact, both instances have the same network complexity index as defined in Kolisch & Sprecher (1997).



4.6(a): Duration Heat Map



4.6(b): Mitigation Frequency



4.6(c): Gantt Chart

Figure 4.6: Result graphics for the j3025_9.sm instance.

5
Robust Optimization for Investment-Cost Tradeoff of a Portfolio of Interdependent Multi-Mode Projects Under Uncertainty

In this chapter, we consider the problem of determining a cost-effective resource allocation on strategies to accelerate activities (mode selection or activity crashing) and strategies to alleviate the potential impact of risks (risk mitigation) to minimize the total cost of a portfolio of interdependent projects. We also adopt a risk-based approach to model the uncertainty that affects the portfolio, with risk events impacting activity durations and costs. Each portfolio project is also subject to tardiness penalties for delayed completions. Within this context, we follow the proposed robust methodology by modeling the uncertain environment as an adversary that selects a worst-case (highest impact) combination of risks given the decision maker's actions. The solution strategy is also developed by following a reformulation scheme that commences from a bi-level (nonlinear) formulation and results in a cut-generation solution algorithm. The next two sections are dedicated to present the problem and the corresponding solution strategy.

5.1
Problem Modeling

5.1.1
Projects, Activities, and the AoN Network

Given a set of interdependent projects P, we represent all project activities (A) and their corresponding precedence relations (E) in an AoN network $G = (A, E)$. Nodes represent the activities and the edges precedence relations. Each activity a is part of a project $p \in P$, with the set A_p representing the activities of the project p. For each activity a is also defined a set of execution modes M_a, with each mode $m \in M_a$ having a corresponding duration d_a^m and cost c_a^m. A precedence relation is defined by predecessor and successor activities i and j respectively, and it is represented here as (i, j). These activities could be part of the same or different projects. For ease of presentation, we only use the previously described zero-lag finish-to-start precedences; however, our models could be easily adapted to handle generalized precedence relations with minimal time-lags by applying the transformations proposed in Bartusch et al. (1988). Each project p has also a due date (τ_p) and an associated tardiness penalty factor by delayed time unit (ρ_p). Given f_p, as the finishing (or completion) time of the project p, the total tardiness penalty for p is determined by the following equation: $max\{0, \rho_p * (f_p - \tau_p)\}$.

In the AoN representation $G = (A, E)$, f_p is equivalent to the longest path from a source dummy activity to the sink dummy activity of project p. As discussed in the previous chapter, the network G is a DAG, so the problem of determining the longest path from source to any other node admits a polynomial-time solution algorithm and a network-flow integer formulation with a totally unimodular constraint matrix (TUM). As Section 5.2 demonstrates, our models take advantage of this TUM formulation. In Section 5.1.4, we provide a detailed example of the AoN network for representing the portfolio.

5.1.2
Risks, Mitigations and the Robust Approach

The uncertainty faced by the projects is represented by a set of discrete risk events (R). The materialization of a risk will affect activity costs and durations. The magnitude of these impacts will depend on the actions taken by the decision maker to minimize them. These actions correspond to the implementation of risk-mitigation plans. Then, we assumed that, for each risk r, the decision maker could implement one of the mitigation plans defined in the set L_r. Given the set of impacted activities by r (I_r), for each mitigation plan l, is defined corresponding duration (Δ^l_{ar}), and cost (θ^l_{ar}) impacts for each $a \in I_r$, and its implementation cost (h^l_r). For ease of presentation, we also assumed that L_r contains the zero-cost plan of taking no action to minimize the risk impacts. The decision of which mitigation plan to implement for a particular risk could be interpreted as a resource allocation decision to define the risk "impact mode" to which the activities will be subject to. Furthermore, we defined an investment budget α, which limits the total resource allocation on activity modes and risk-mitigation plans.

To account for the uncertainty, we used a robust optimization approach inspired in the work of Bertsimas & Sim (2004). In our approach, the environment acts like an adversary, which selects the worst-case feasible combination of risks that maximizes the impact on the total cost of the portfolio for any given action plan. The feasibility criteria for the combination of risks (risk-scenarios) entails the user conservatism over the environment and defines the search space for the adversary (i.e., the uncertainty set). Let σ_r^l define the cost that the adversary incurs for materializing a risk r when this risk is in impact mode l (i.e., given that risk mitigation plan l was implemented). The total uncertainty budget for the adversary is β. Moreover, the values for σ_r^l should be such that they are higher for relative low-probability risks and lower for the relative high-probability ones. Thus, the adversary is limited to choosing a combination of risks for which the sum of the corresponding σ_r^l does not exceed β. An increase on β also means an increase on the user's conservatism over the environment, as the adversary will expand its capacity of risk materialization. In our model, the amount deducted from the uncertainty budget for the adversary to materialize a particular risk depends on the chosen mitigation action. This approach accounts for the decision-dependent uncertainty aspect of the problem, where a mitigation plan may not only minimize risk impacts but may also decrease its chance of materializing.

5.1.3 Problem Statement

The problem is stated as follows: given a set of interdependent projects P, with corresponding multi-mode activities A and precedence relations E, and a set of uncertain risk events R faced by P, with corresponding mitigation options L, find the best combination of activities' modes and mitigation plans to be implemented (not exceeding α) that minimizes P's total cost (i.e., sum of activities' costs, mitigations' costs, and tardiness penalties) of the worst-case feasible risk scenario that could be materialized by the adversary.

5.1.4 Example

A hypothetical energy company decided to install a new thermal power plant. To manage this objective, the company developed two projects: a construction (CP) and an environmental project (EP). The CP controls the activities related to engineering, construction, and operations startup, while the EP manages all activities related to environmental licenses and requirements. Furthermore, the CP has an associated penalty for delays in delivering the committed energy with their contractors. Its due date is $\tau_{CP} = 20$ and the penalty is $\rho_{CP} = 10$. The EP also has an associated penalty for delays in fulfilling the environmental compensations committed with the governmental agency ($\tau_{EP} = 17$ and $\rho_{EP} = 3$).

In Table 5.1, we detail the activities and their associated data. Each activity has a baseline mode that maps the lowest cost longest duration

Proj.	*Activity, a*	*Description*	d_a^1	c_a^1	d_a^2	c_a^2	*Pred.*
CP	Engineering, E	Engineering Phase	5	5	-	-	-
CP	Construction, C	Construction Phase	10	10	7	20	{E, IL}
CP	Operations, O	Commissioning and start of operations	3	3	-	-	{C, OL}
EP	Installation Licensing, IL	Process of obtaining the environmental license to start the construction phase	1	1	-	-	-
EP	Implementation of Environmental Compensation, IEC	Execute the required environmental compensation projects (e.g., improve infrastructure of nearby cities)	7	7	5	14	{IL}
EP	Operations Licensing, OL	Process of obtaining the operating license to start operations	2	2	-	-	{IEC}

Table 5.1: Activities data.

mode, which is referred to as mode 1 in Table 5.1. The activities related to construction (C and IEC) have a crashing mode that reflects the use of extra workers to decrease their durations. This is called mode 2 in Table 5.1. In Table 5.2, we describe the main risks and their mitigation plans identified by the company. Each risk r has two impact modes $L_r = \{1, 2\}$. Mode 1 maps the uncertainty impacts of the decision of not implementing the associated mitigation plan (i.e., $h_r^1 = 0$), while mode 2 maps the impacts in the case of implementing the mitigation. In Table 5.3, the associated uncertainty impacts are provided.

Risk, r	*Description*	*Mitigation Plan, l=2*	I_r	h_r^2	σ_r^1	σ_r^2
New Environmental Regulations, R1	Chance of approval of a new environmental law	Implement in advance the requirements of the possible new law	{IL, IEC, OL}	10	40	70
Strike, R2	Chance of strike of construction workers	Negotiate a bonus for a successful project delivery	{C, IEC}	15	50	70
Critical Equipment Failure, R3	Chance of failure in critical equipments (e.g., generators) during commissioning	Buy high quality equipments	{O}	18	60	90

Table 5.2: Risks data.

The company has a total investment budget of $\alpha = 68$ and is adopting a limited uncertainty budget for the adversary of $\beta = 110$. Given these definitions, we present two scenarios of decisions. In the scenario illustrated

Risk, r	*Imp. Act., a*	Δ^1_{ar}	θ^1_{ar}	Δ^2_{ar}	θ^2_{ar}
R1	IL	1	1	0	0
R1	IEC	4	4	2	2
R1	OL	2	2	0	0
R2	C	5	5	2	2
R2	IEC	3	3	1	1
R3	O	6	6	6	6

Table 5.3: Uncertainty impacts data.

in Figure 5.1, the decision maker decided to maintain the original plan of all activities in their baseline modes and to not implement mitigations. The second scenario, detailed in Figure 5.2, illustrates the optimal investment decision. The figures detail the associated worst-case scenario determined by the adversary along with all the costs that compose the total portfolio cost. The *AoN* network is also represented to demonstrate the penalties calculations. In the network, we present the source dummy activity 0 that is connected with all activities with no predecessors, while the project dummy sink nodes (s_{CP} and s_{EP}) are connected with the corresponding project activities that have no successors. The edges represent the precedence relations, and their weights are equivalent to their corresponding predecessors' durations. In a number of edges, the weights are presented as sums; this representation reflects the mode duration summed with the duration impact of a materialized risk. In the first scenario, the portfolio's total cost is 167, while in the optimal scenario it is 99, emphasizing the importance of an effective resource allocation decision in the proposed problem.

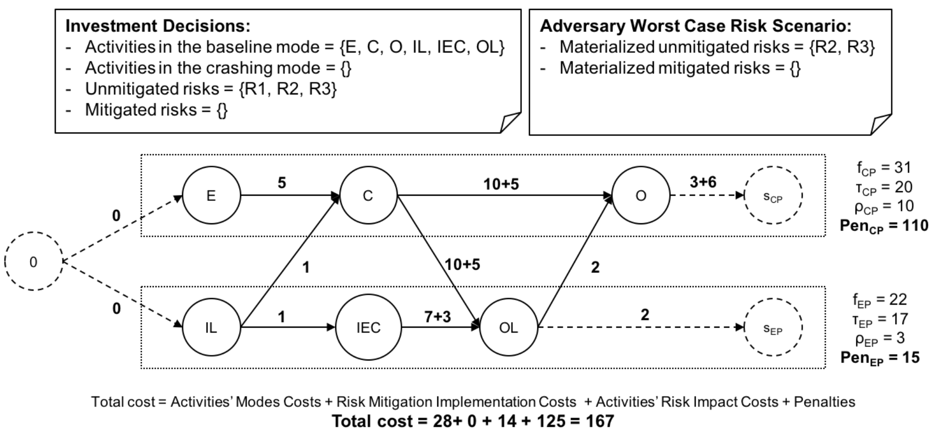


Figure 5.1: Baseline investment scenario.

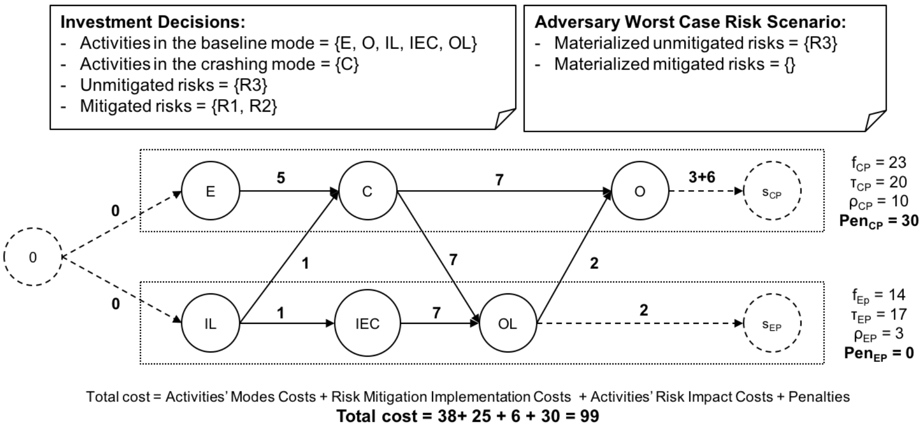


Figure 5.2: Optimal investment scenario.

5.2 Mathematical Formulations and Cut-Generation Algorithm

5.2.1 Min-Max Model

We present a min-max bi-level formulation that assumes non-negativity of all parameters. The first level models the decision maker's actions, i.e. the selection of activity modes (x) and risk-mitigation plans (y). The second level models the adversary's actions, i.e. the selection of the worst-case feasible risk scenario (z) for the corresponding first-level decisions x and y. Considering the definitions presented below, the model follows:

x_a^m	binary decision variable that indicates the selection of mode m for the activity a by the decision maker.
y_r^l	binary decision variable that indicates the implementation of the mitigation plan l for the risk r by the decision maker.
$\Pi(x, y)$	function that given x and y decisions, returns the value of the maximum cost impact of a feasible risk-scenario. This function represents the objective value of the adversary's action (i.e., the second level objective).
α	investment budget.

$$\underset{x,y}{\text{Min}} \quad \sum_{a \in A} \sum_{m \in M_a} c_a^m x_a^m + \sum_{r \in R} \sum_{l \in L_r} h_r^l y_l^r + \Pi(x, y) \tag{5-1}$$

$s.t.$

$$\sum_{a \in A} \sum_{m \in M_a} c_a^m x_a^m + \sum_{r \in R} \sum_{l \in L_r} h_r^l y_l^r \leq \alpha \tag{5-2}$$

$$\sum_{m \in M_a} x_a^m = 1 \quad \forall a \in A \tag{5-3}$$

$$\sum_{l \in L_r} y_r^l = 1 \quad \forall r \in R \tag{5-4}$$

$$x_a^m \in \{0,1\} \quad \forall a \in A, m \in M_a \tag{5-5}$$

$$y_r^l \in \{0,1\} \quad \forall r \in R, l \in L_r \tag{5-6}$$

The model's objective is to decide the best combination of activity modes (x) and risk-mitigation plans (y), which minimizes the total cost of the portfolio P under the assumption that the feasible risk scenario with maximum cost impact will unfold. The first and second terms of the objective function (5-1) account for the costs of activity modes and risk-mitigation plans respectively, whereas the last term accounts for the cost impact due to the uncertainty, that is, the cost impact of the adversary's decision. Constraint (5-2) limits the resource (or investment) that could be allocated to activity modes and mitigation plans. This constraint is removed in the case of an unlimited budget α, then the resulting model will represent the optimal trade-off between investment and total cost. Constraints (5-3) guarantee that one activity mode is selected for each activity, while constraints (5-4) ensure that one mitigation plan is implemented for each risk. To complete our min-max model definition, we present an optimization model that is equivalent to the second-level decision represented by adversary's function $\Pi(x, y)$. Given first-level decisions x and y, the model aims at selecting the best combination of risks (z), not exceeding the uncertainty budget, that maximizes the cost impact due to tardiness penalties and risk impacts on the activity costs. To account for the tardiness penalties, the model has to determine projects' completion times, which is achieved by using extra network-flow variables and constraints. Next, the required extra definitions are presented along with the equivalent optimization model $\Pi(x, y)$:

z_r^l	binary decision variable that indicates the selection (or materialization) of the risk r on the "impact mode" due to the mitigation l.
o_p	decision variable that accounts for the time overrun (delay) of project p.
f_p	decision variable that accounts for the finishing (or completion) time of project p.
v_p	binary decision variable that indicates a due date violation for the project p.

u_{ij}^{p} decision variable that indicates if the precedence arc (i,j) is part of the longest path from the artificial source node to the project's p dummy sink node s_p.
zu_{rl}^{ijp} decision variable that represents the product $z_r^l \cdot u_{ij}^p$.
R_a set of risks that impact activity a (i.e., if $a \in I_r$, then $r \in R_a$)
MDO constant that accounts for the maximum possible due date overrun, which plays the role of a *big-M* or "infinity" constant. The sum of all activity durations and corresponding risk impacts is an example of a possible loose setting for MDO (i.e., $\sum_{a\in A}[\sum_{m\in M_a} d_a^m + \sum_{r\in R_a}\sum_{l\in L_r}\Delta_{ar}^l]$).
δ_a^- set of predecessors of activity a ($\delta_a^- = \{i \mid (i,a) \in E\}$).
δ_a^+ set of successors of activity a ($\delta_a^+ = \{j \mid (a,j) \in E\}$).

$$\Pi(x,y) = \underset{z,u,zu,f,o,v}{\text{Max}} \sum_{p\in P}\rho_p o_p + \sum_{a\in A}\sum_{r\in R_a}\sum_{l\in L_r}\theta_{ar}^l z_r^l \tag{5-7}$$

$s.t.$

$$z_r^l \le y_r^l \quad \forall r \in R, l \in L_r \tag{5-8}$$

$$\sum_{r\in R}\sum_{l\in L_r}\sigma_r^l z_r^l \le \beta \tag{5-9}$$

$$f_p = \sum_{(i,j)\in E}\Big[\sum_{m\in M_i} d_i^m x_i^m u_{ij}^p + \sum_{r\in R_i}\sum_{l\in L_r}\Delta_{ir}^l zu_{rl}^{ijp}\Big] \;\forall p \in P \tag{5-10}$$

$$\sum_{j\in\delta_i^+} u_{ij}^p = 1 \quad \forall p \in P,\ i = 0 \tag{5-11}$$

$$\sum_{i\in\delta_j^-} u_{ij}^p = 1 \quad \forall p \in P,\ j = s_p \tag{5-12}$$

$$\sum_{i\in\delta_a^-} u_{ia}^p - \sum_{j\in\delta_a^+} u_{aj}^p = 0 \quad \forall p \in P,\ a \in A,\ a \notin \{0, s_p\} \tag{5-13}$$

$$o_p \le f_p - \tau_p v_p \quad \forall p \in P \tag{5-14}$$

$$o_p \le MDO \cdot v_p \quad \forall p \in P \tag{5-15}$$

$$zu_{rl}^{ijp} \le z_r^l \quad \forall p \in P, (i,j) \in E, r \in R_i, l \in L_r \tag{5-16}$$

$$zu_{rl}^{ijp} \le u_{ij}^p \quad \forall p \in P, (i,j) \in E, r \in R_i, l \in L_r \tag{5-17}$$

$$zu_{rl}^{ijp} \ge z_r^l + u_{ij}^p - 1 \quad \forall p \in P, (i,j) \in E, r \in R_i, l \in L_r \tag{5-18}$$

$$z_r^l \in \{0,1\} \quad \forall r \in R, l \in L_r \tag{5-19}$$

$$u_{ij}^p \in \{0,1\} \quad \forall (i,j) \in E, p \in P \tag{5-20}$$

$$v_p \in \{0, 1\} \quad \forall p \in P \tag{5-21}$$

$$f_p \geq 0 \quad \forall p \in P \tag{5-22}$$

$$o_p \geq 0 \quad \forall p \in P \tag{5-23}$$

The first term of the objective function (5-7) accounts for the tardiness penalties for each project, while the second accounts for the activity cost impacts due to the materialized risks. Constraints (5-8) ensure that the corresponding risk impacts will take into account the mitigation plan selected on the first-level decision, while constraint (5-9) guarantees that the selected risks do not exceed the uncertainty budget.

Constraints (5-10), (5-11), (5-12), and (5-13) are responsible for the calculation of project's finishing times (f) through a multi-commodity network-flow strategy that determines the longest paths from source node (0) to milestone sink nodes ($s_p, \forall p \in P$). Each project p has its own commodity flow (or path), which starts at the source node (0), due to constraints (5-11), flows through the network edges and nodes, due to flow-conservation constraints (5-13), and arrives at the corresponding sink nodes (s_p), due to constraints (5-12). The path sizes (or project's finishing times) from source to sink nodes are determined by constraints (5-10), they are equivalent to the sum of edge weights that are part of the longest path. As detailed on Section 5.1.4, the weight of an edge (i, j) is equal to the duration of its predecessor activity (i), which, in our problem, is dependent of its mode duration and the materialized risk duration impacts. Moreover, the risk impact on the activity duration is only accounted for when the risk r, under the mitigation l, is materialized by the adversary ($z_r^l = 1$), which forces the use of the linearization variable zu_{rl}^{ijp} on the edge's weight expression. The linearization $zu_{rl}^{ijp} = z_r^l \cdot u_{ij}^p$ is guaranteed by constraints (5-16), (5-17), and (5-18).

Constraints (5-14) and (5-15) are responsible for projects' delays calculations (i.e., $o_p = max\{0, f_p - \tau_p\}$). The binary variable v_p indicates a due date violation for the project p, given that o_p is being maximized on (5-7), when $f_p < \tau_p$, the best choice, on the objective function perspective, is to set $v_p = 0$, allowing also o_p to be zero. In the opposite case, when $f_p \geq \tau_p$, the best choice is to set $v_p = 1$, allowing o_p to be set with the proper delay $f_p - \tau_p$.

In its current form, it might be difficult to solving the above-mentioned problem. To circumvent this difficulty, we performed a series of transformations and present a cut-generation solution algorithm.

5.2.2
Risk Scenarios Enumeration Model

The first step toward the solution algorithm is to transform the model into a one-level model that directly incorporates the adversary's decision. The main idea of this transformation is to enumerate all possible feasible risk scenarios and to directly account for their corresponding objective values in the model. We considered a feasible risk scenario (ξ), an assignment of values for the adversary's decision variables z_r^l, which respects constraints (5-9), (5-19) and $\sum_{l \in L_r} z_r^l \leq 1, \forall r \in R$. Next, we extended our definitions to present the one-level formulation in sequence:

RS	set of all feasible risk scenarios.
ξ	a feasible risk scenario (i.e., $\xi \in RS$).
T	auxiliary variable to account for the maximum cost impact over all $\xi \in RS$, which is equivalent to the value of the function $\Pi(x, y)$.
z_{rl}^{ξ}	constant with the value of z_r^l in the risk scenario $\xi \in RS$.
LP	or $LP(x, y, \xi, p)$, a function that, given the decision maker's decisions x and y, and a risk scenario ξ (i.e., adversary's decision), returns the corresponding size of the longest path from the source node (0) to the project's sink node s_p.
f_p^{ξ}	auxiliary variable that accounts for project's p finishing (or completion) time in the risk scenario ξ.
o_p^{ξ}	auxiliary variable that accounts for project's p total time overrun (delay) in the risk scenario ξ.

$$\min_{x,y,o,f} \sum_{a \in A} \sum_{m \in M_a} c_a^m x_a^m + \sum_{r \in R} \sum_{l \in L_r} h_r^l y_l^r + T \tag{5-24}$$

$s.t.$

$$T \geq \sum_{p \in P} \rho_p o_p^{\xi} + \sum_{a \in A} \sum_{r \in R_a} \sum_{l \in L_r} \theta_{ar}^l z_{rl}^{\xi} y_r^l \quad \forall \xi \in RS \tag{5-25}$$

$$f_p^{\xi} = LP(x, y, \xi, p) \quad \forall \xi \in RS, \; p \in P \tag{5-26}$$

$$o_p^{\xi} \geq f_p^{\xi} - \tau_p \quad \forall \xi \in RS, \; p \in P \tag{5-27}$$

$$o_p^{\xi} \geq 0 \quad \forall \xi \in RS, \; p \in P \tag{5-28}$$

$$(5-2), (5-3), (5-4), (5-5) \text{ and } (5-6)$$

Constraints (5-25) are equivalent to a maximum between risk scenario cost impacts (given x and y). The cost impact of a risk scenario (ξ) is composed by the penalties due to the project delays and the cost impact on activities. The cost impact on activities is only considered for the risks under the mitigation selected by the decision maker (i.e., $y_r^l = 1$). The project delays for each risk scenario (o_p^ξ) are determined by the constraints (5-26) and (5-27). Given the materialized risks of a scenario ($z_{rl}^\xi = 1$) and the x (activity modes) and y (mitigation plans) decisions, we completely define the activities' final durations. As such, the function LP uses these durations to assign network's final edge weights and, subsequently, calculate the corresponding longest path from source to the project's sink node s_p. We replaced this function during next steps, yet, for now, it is enough to understand its role. With projects' finishing times by scenario (f_p^ξ), we use constraints (5-27) to properly determine their corresponding delays (o_p^ξ). Using the fact that this new formulation is a minimization and o_p^ξ is also being minimized, when $f_p^\xi < \tau_p$, the greater than zero constraint (5-28) will force $o_p^\xi = 0$, while in the case that $f_p^\xi \geq \tau_p$, constraint (5-27) will force $o_p^\xi = f_p^\xi - \tau_p$.

5.2.3 Risk Scenarios Enumeration Model with Network-Flow Dual Constraints

We used a network-flow formulation to define the optimization model that is equivalent to the function LP. It is a longest-path model with a proper objective function that assigns the network's edge weights depending on activity modes (x), implemented risk mitigations (y), and the risk scenario (ξ). Given u_{ijp}^ξ as the network-flow variable for the arc (i,j), project p, and risk-scenario ξ, we formulated the equivalent network-flow model for the LP function as follows:

$$LP(x, y, \xi, p) =$$

$$\operatorname*{Max}_{u} \sum_{(i,j)\in E} \Big[\sum_{m\in M_i} d_i^m x_i^m + \sum_{r\in R_i} \sum_{l\in L_r} \Delta_{ir}^l z_{rl}^\xi y_r^l \Big] u_{ijp}^\xi \tag{5-29}$$

$$s.t.$$

$$\sum_{j\in\delta_i^+} u_{ijp}^\xi = 1 \quad i = 0 \tag{5-30}$$

$$\sum_{i\in\delta_j^-} u_{ijp}^\xi = 1 \quad j = s_p \tag{5-31}$$

$$\sum_{i\in\delta_a^-} u_{iap}^\xi - \sum_{j\in\delta_a^+} u_{ajp}^\xi = 0 \quad \forall a \in A,\ a \notin \{0, s_p\} \tag{5-32}$$

$$0 \leq u_{ijp}^\xi \leq 1 \quad \forall (i,j) \in E \tag{5-33}$$

As already mentioned, in our AoN network representation, in the objective function, the weight of an edge $(i,j) \in E$ is equal to the duration of its predecessor activity i. In our approach, the final duration of an activity depended on its mode duration and the materialized risks that affect it. The expression, which is multiplying variables u^{ξ}_{ijp}, on the objective function (5-29), reflects exactly the final duration of the predecessor activity i as the edge's weight of $(i, j) \in E$. Taking advantage of the total unimodularity of the network-flow constraint matrix, we use a linear model, since its optimal solution is equivalent to its binary version (Papadimitriou & Steiglitz, 1982). By the strong-duality property of the linear programming models, the objective of the optimal dual solution is equal to the objective of the optimal primal solution (see Papadimitriou & Steiglitz (1982)); therefore, we could also use the dual version of the presented model as the model to represent the function LP. The dual version of the longest-path problem has also been used in Artigues et al. (2013) for a robust version of the RCPSP. Next, we present this dual version:

π^{ξ}_{ap}	decision variable due to the primal network-flow constraint on activity a, for project p and risk scenario ξ.
γ^{ξ}_{ijp}	decision variable due to primal greater than zero constraint on network arc (i, j), for project p and risk scenario ξ.

$$LP(x, y, \xi, p) = \min_{\pi,\mu} \quad \pi^{\xi}_{0p} + \pi^{\xi}_{s_p p} + \sum_{(i,j)\in E} \gamma^{\xi}_{ijp} \tag{5-34}$$

s.t.

$$\pi^{\xi}_{jp} - \pi^{\xi}_{ip} + \gamma^{\xi}_{ijp} \geq \sum_{m\in M_i} d^m_i x^m_i + \sum_{r\in R_i}\sum_{l\in L_r} \Delta^l_{ir} z^{\xi}_{rl} y^l_r \quad \forall (i, j) \in E, i \neq 0 \tag{5-35}$$

$$\pi^{\xi}_{jp} + \pi^{\xi}_{0p} + \gamma^{\xi}_{0jp} \geq 0 \quad \forall j \in \delta^+_0 \tag{5-36}$$

$$\gamma^{\xi}_{ijp} \geq 0 \quad \forall (i, j) \in E \tag{5-37}$$

Constraints (5-36) relate to the outgoing arcs from the source node, and constraints (5-35) relate to the other network arcs.

Using the approach in Bertsimas & Sim (2004), we incorporated this dual model, which is equivalent to the LP function, directly into the previous problem model:

$$\min_{x,y,o,f,\pi,\gamma} \quad \sum_{a\in A}\sum_{m\in M_a} c^m_a x^m_a + \sum_{r\in R}\sum_{l\in L_r} h^l_r y^r_l + T \tag{5-38}$$

s.t.

$$T \geq \sum_{p\in P} \rho_p o^{\xi}_p + \sum_{a\in A}\sum_{r\in R_a}\sum_{l\in L_r} \theta^l_{ar} z^{\xi}_{rl} y^l_r \quad \forall \xi \in RS \tag{5-39}$$

$$f_p^{\xi} = \pi_{0p}^{\xi} + \pi_{s_p p}^{\xi} + \sum_{(i,j)\in E} \gamma_{ijp}^{\xi} \quad \forall \xi \in RS,\ p \in P \tag{5-40}$$

$$\pi_{jp}^{\xi} - \pi_{ip}^{\xi} + \gamma_{ijp}^{\xi} \geq \sum_{m\in M_i} d_i^m x_i^m + \sum_{r\in R_i} \sum_{l\in L_r} \Delta_{ir}^l z_{rl}^{\xi} y_r^l \qquad \forall (i,j) \in E,\ i \neq 0, \xi \in RS,\ p \in P \tag{5-41}$$

$$\pi_{jp}^{\xi} + \pi_{0p}^{\xi} + \gamma_{0jp}^{\xi} \geq\ 0 \quad \forall j \in \delta_0^+, \quad \forall \xi \in RS,\ p \in P \tag{5-42}$$

$$o_p^{\xi} \geq f_p^{\xi} - \tau_p \quad \forall \xi \in RS,\ p \in P \tag{5-43}$$

$$o_p^{\xi} \geq 0 \quad \forall \xi \in RS,\ p \in P \tag{5-44}$$

$$\gamma_{ijp}^{\xi} \geq\ 0 \quad \forall (i,j) \in E, \quad \forall \xi \in RS,\ p \in P \tag{5-45}$$

$$(5-2), (5-3), (5-4), (5-5) \text{ and } (5-6)$$

This is an MIP model that could be solved directly by commercially available solvers. A difficulty that arises here stems from the fact that the model's size grows exponentially with the number of risks ($|R|$). Hence, a cut-generation algorithm, similar to the one presented in the Chapter 4, is proposed to deal with this characteristic of the model.

5.2.4 Cut-Generation Algorithm

The idea of the algorithm is to only maintain a subset of the risk-scenarios ($SRS \subseteq RS$) in the master problem. A separation procedure is called at each iteration to identify whether a risk scenario not in SRS should be considered. In this case, the scenario is included in the master problem and the algorithm proceeds. Otherwise, the x and y obtained by solving the master problem are optimal. The separation problem is the optimization problem equivalent to the function $\Pi(x, y)$. The solution of this problem will return the worst-case risk scenario represented by the values of variables z_r^l. The Algorithm 3 outlines the cut-generation solution strategy in greater detail.

Algorithm 3 Robust Risk-Mitigation Cut-Generation

$SRS \leftarrow \emptyset$ ▷ SRS initialization
$\xi \leftarrow \emptyset$ ▷ No Risk Scenario
$LB \leftarrow 0$ ▷ Lower Bound initialization
$UB \leftarrow \infty$ ▷ Upper Bound initialization
$OptSol \leftarrow \emptyset$ ▷ Optimal Solution initialization
$MP \leftarrow$ Create master problem from the model of Section 5.2.3
while $LB < UB$ **and** $ElapsedTime < TimeLimit$ **do**
  Add ξ to SRS and its corresponding cut to MP (variables and constraints related to ξ of the model of Section 5.2.3)
  $MPSol \leftarrow$ Solve MP to obtain its current solution
  $LB \leftarrow MPSol.obj$ ▷ LB is updated with the objective value of $MPSol$
  if $LB = UB$ **then**
    break
  end if
  $x \leftarrow MPSol.x$ ▷ Get x decisions from $MPSol$
  $y \leftarrow MPSol.y$ ▷ Get y decisions from $MPSol$
  $SP \leftarrow$ Create separation problem for current x and y according with the model of $\Pi(x, y)$
  $SPSol \leftarrow$ Solve SP to obtain its current solution
  $\xi \leftarrow SPSol.z$ ▷ Get current worst-case risk scenario from $SPSol$
  $Investment \leftarrow \sum_{a \in A} \sum_{m \in M_a} c_a^m x_a^m + \sum_{r \in R} \sum_{l \in L_r} h_r^l y_l^r$ ▷ Total investment of x and y decisions
  $UncCostImpact \leftarrow SPSol.obj$ ▷ Uncertainty cost impact (i.e., $\Pi(x, y)$)
  $TotalPortCost \leftarrow Investment + UncCostImpact$ ▷ Total portfolio cost
  if $TotalPortCost < UB$ **then**
    $OptSol \leftarrow x, y$
    $UB \leftarrow TotalPortCost$
  end if
end while
return $OptSol$

5.3 Computational Experiments

Managing a portfolio of interdependent projects is a complex task. Due to this complexity, these projects are often managed independently by, for instance, different staff, divisions, or even companies. As a result, decisions are often taken for each project separately, ignoring the interdependencies between them. Relying on this fact, we designed a set of experiments, detailed in Section 5.3.2, to highlight the importance of an integrated decision making for effective risk mitigation for a portfolio of interdependent projects. A second set of experiments, elucidated in Section 5.3.2, was designed to assess the performance of the proposed solution strategy when subject to variations of the problem's main dimensions, which are the number of activities and risks. These experiments also aimed at evaluating the computational feasibility, robustness and quality of the proposed approach. In Section 5.3.1, we describe the methodology applied to generate the set of instances used on the computational experiments.

5.3.1 Instances

The project portfolios of the experiments' instances were formed by randomly selecting (without repetition) 4 projects that were generated based on instances of the multi-mode resource constrained project scheduling problem (MRCPSP) from *PSPLIB* (Kolisch & Sprecher, 1997). Each of these projects is generated by combining activities, precedence relations, and tardiness cost (ρ_p) from a particular MRCPSP instance with additional input data required by our model. Next, we present the generation of this required data for each project and explain how these projects were integrated in different ways in each portfolio.

Activity Modes

For each non-dummy activity (a) we defined two different modes. The first mode, which has lower cost and longer duration, is defined to map the baseline planning of the activity. Its duration (d_a^1) is equal to the longest duration from the corresponding activity modes defined in the MRCPSP instance. Its cost is set to be equal to its duration (i.e., $c_a^1 = d_a^1$), which reflects the assumption that the cost of the resource performing the activity is one unit per time unit. The second mode maps the assumption where the decision maker doubles the resources when performing the activity. Then, this mode costs the double of the first mode (i.e., $c_a^2 = 2 \cdot c_a^1$) and its duration is sampled from a discrete uniform distribution. Given $DU(a, d)$, as the representation of a discrete uniform distribution in the interval $[a, b]$, the second mode duration d_a^2 was sampled from $DU(\lfloor \frac{d_a^1}{2} \rfloor, d_a^1 - 1)$, that is, from a maximum resource effectiveness that is half of first mode duration to a minimum one that is only one time unit less than first mode duration.

Risks and Mitigations

For each project, risks were generated having two "modes" (i.e., $L_r = \{1, 2\}$) . The first mode (risk mode) represents the case in which no mitigation action is taken to reduce risk impacts, while the second mode (mitigation mode) maps a version of the risk when a particular mitigation plan is implemented. The set of impacted activities by a risk r (I_r) was generated by randomly selecting 3 activities of the same project. This was adopted only to facilitate our analysis, the proposed method does not have this limitation, the same risk may impact multiple activities from different projects. In what follows, we describe how each impact mode was generated.

As expected, the cost of the first impact mode was set to zero (i.e., $h_r^1 = 0$). Despite not using risk probabilities directly in our method, we used this concept here to help describe the rationale applied to generate model input parameters associated with risks. We started by first generating a risk probability $Prob_r$ value by sampling from a uniform continuous distribution ($U(a,b)$). Its minimum (a) and maximum (b) parameters were defined according to a randomly selected "probability category" as detailed in Table 5.4. As σ_r^l values reflect the difficulty of the "adversary" to materialize a risk, it has an inverse relation with the risk probability. Then, to map this relation, we employed the following equation to generate the corresponding value for the first mode: $\sigma_r^1 = 100 \cdot (1.0 - Prob_r)$. To generate duration impacts for each impacted activity (i.e., Δ_{ar}^1, where $a \in I_r$), we first randomly selected an "impact category" for the risk, and then sampled for each activity a value from the corresponding discrete uniform distribution (see Table 5.5). The cost impact for each activity a (i.e., θ_{ar}^1, where $a \in I_r$) was generated by multiplying the corresponding duration impact Δ_{ar}^1 with a random resource allocation cost by time unit sampled from $DU(\frac{c_a^1}{d_a^1} = 1, \lceil \frac{c_a^2}{d_a^2} \rceil)$. These limits are simply the cost by time unit of the two activity modes.

Prob. Cat.	**Min Prob. (a)**	**Max Prob. (b)**
Low	0.05	0.35
Medium	0.2	0.4
High	0.4	0.7

Table 5.4: Risk probability distribution parameters for risk mode.

Imp. Cat.	**Min Imp. (a)**	**Max Imp. (b)**
Low	1	3
Medium	3	7
High	6	10

Table 5.5: Duration impact distribution parameters for risk mode.

To generate the second (or mitigation) mode of a risk r, we first randomly selected, with equal probability, an effectiveness factor for reducing risk probability ($Fprob_r$) and an effectiveness factor for reducing risk impacts ($Fimp_r$). These factors were sampled with equal probability from the set

$\{0.1, 0.5, 0.7\}$. The $Fprob_r$ factor was applied to reflect a reduction in the risk probability $Prob_r$ after the mitigation is implemented (i.e., $ProbAfterMit_r = Fprob_r \cdot Prob_r$). This reduction was mapped in a higher σ value for the mitigation mode determined by the following equation: $\sigma_r^2 = 100 \cdot (1.0 - ProbAfterMit_r)$. The $Fimp_r$ factor was also applied as a reduction factor, in this case, for the risk impacts in duration and cost of the activities in I_r (i.e., $\Delta_{ar}^2 = Fimp_r \cdot \Delta_{ar}^1$ and $\theta_{ar}^2 = Fimp_r \cdot \theta_{ar}^1$, $\forall a \in I_r$). The implementation cost of the mitigation mode h_r^2 was generated by multiplying the expected value of cost impact reduction (i.e., $EcostImpRed_r = Prob_r \cdot \sum_{a \in I_r} \theta_{ar}^1 - ProbAfterMit_r \cdot \sum_{a \in I_r} \theta_{ar}^2$) with a randomly selected cost factor ($Fcost_r$) sampled from the set $\{2.0, 2.5, 3.5\}$ (i.e., $h_r^2 = Fcost_r \cdot EcostImpRed_r$).

Portfolio Topologies and Project Deadlines

As mentioned in the beginning of this section, we formed project portfolios by grouping four different projects generated according to the previously described methodology. To integrate these projects at a particular portfolio, we used four different topologies of interdependencies between them. Figure 5.3 presents these topologies, where each node represents a project from the portfolio and each edge from a project p_1 to a project p_2 represents a finish-to-start constraint between those projects. This constraint was achieved by adding a finish-to-start precedence relation from p_1's sink activity to p_2's source activity. These topologies were proposed to analyze how decisions are influenced by the degree of interdependency between the projects of the portfolio. We refer to the interdependency in the sense of the range of direct or indirect impact effect of a risk. For instance, if a risk of the project A materializes, while in the Full-Parallel (FP) topology it will only impact itself, in the Sequential (SQ) topology, it will also potentially impact projects B, C, and D. In that sense, the degree of interdependency increases in the following sequence: Full-Parallel (FP), Half-Parallel (HP), Diamond (DD) and Sequential (SQ).

The only missing parameters to be described are the project due dates. These parameters were set to the baseline durations of each project at each portfolio and topology. Hence, for each combination of portfolio and topology, we first calculated the earliest schedule of the activities in the corresponding portfolio network, assuming all activities in their baseline modes. Then, for each project p, we assigned the earliest-finish of its sink activity (s_p) as due date τ_p, which is essentially the project baseline finish at that particular portfolio and topology.

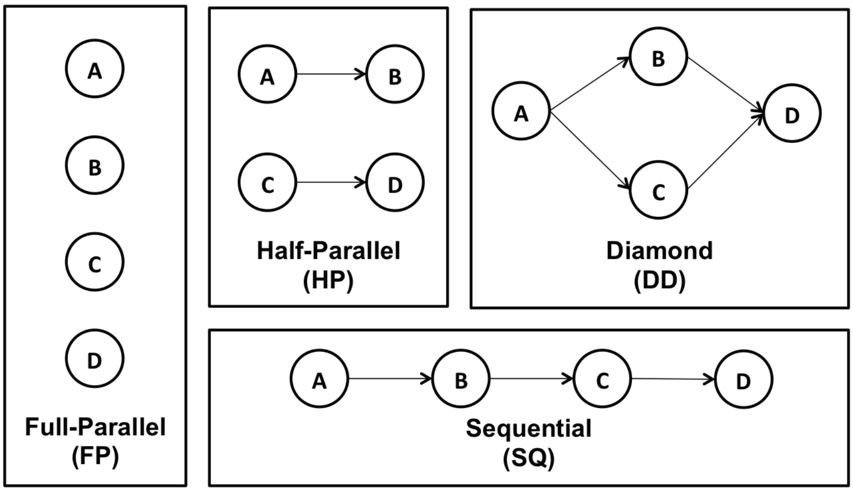


Figure 5.3: Portfolio topologies.

5.3.2 Experiments and Results

All tests were performed on a PC with an Intel(R) Xeon(R) E5-2643 CPU with 2 cores of 3.3GHz, 251 GB of RAM and running Fedora 21 (Twenty One) OS. The code was implemented using Python 2.7.8 and the mathematical models were solved by IBM(R) ILOG(R) CPLEX(R) 12.5.0.0. We also set the $TimeLimit$ of Algorithm 3 to one hour for all experiments.

Effectiveness of Integrated Decisions

For this set of experiments, the projects that compose the portfolios were generated from the MRCPSP instances of the `j10.mm` set. This set contains instances with projects having 10 activities (or jobs), each one also having extra source and sink dummy activities (Section 5.1.4 details the role of these activities). For each project a set of 3 risks was generated. A total of 100 portfolios were formed by combining these projects, and each of these portfolios had a total of 40 non-dummy activities and 12 risks. In these experiments, the adversary uncertainty budget (β) varied between four different levels, with corresponding values equal to 40%, 60%, 80% and 100% of $\sum_{r \in R} \sigma_r^2$. For each portfolio, topology and uncertainty budget, three different solution strategies were tested.

The first strategy was to solve the corresponding global portfolio problem with the proposed cut-generation algorithm. In this strategy, the investment budget (α) was unlimited, so its corresponding solution represented the optimal trade-off between investments and the portfolio total cost, that is, the optimal integrated global decision. In the results, we refer to this strategy as min-max.

The second strategy was implemented to emulate the scenario where decisions are taken independently regarding each project of the portfolio. In this strategy, the investment budget (α) was also unlimited. For each project, we created an individual separate problem that was solved to optimality with the proposed cut-generation algorithm. In the context of each separate project problem, the uncertainty budget level was defined as a percentage of the sum $\sum_{r \in R} \sigma_r^2$ considering only the subset of risks of the corresponding project. In addition, projects' due dates were defined as their baseline durations (i.e., project duration considering all activities in their baseline modes and ignoring inter-project precedences). After separately solving the problems of each project, we combined all the activity modes (x) and risk mitigation decisions (y) in a portfolio solution. The portfolio total cost (or objective value) for this solution was determined by summing the objective of the adversary problem in the integrated network ($\Pi(x, y)$) with the investment in activity modes and risk mitigations. For ease of presentation, we represent this objective function calculation as follows:

$$Obj(x, y) = \sum_{a \in A} \sum_{m \in M_a} c_a^m x_a^m + \sum_{r \in R} \sum_{l \in L_r} h_r^l y_l^r + \Pi(x, y) \tag{5-46}$$

Notice that in this strategy, the portfolio solution (combined x and y of individual project solutions) is the same for all topologies, as it ignores the inter-project precedences, while the objective $Obj(x, y)$ is topology dependent, as $\Pi(x, y)$ is the determined in the integrated portfolio network. Another important fact is that even in the FP topology, the integrated objective evaluation is different from summing the individual objectives of the problems of each project, as the adversary has a limited uncertainty budget in all cases. In the integrated evaluation, the adversary could freely distribute its uncertainty budget for the corresponding level across the risks of all projects to maximize the global impact. In the case where the problems are solved individually by each project, the allocation of the uncertainty budget across projects was fixed and defined by the corresponding level. As this strategy maps a limitation on how companies often take their decisions, it is also a comparison baseline for our proposed method. We refer to this strategy as independent decisions.

As companies have budget limitations to invest in their projects, the final strategy was to solve the integrated portfolio problem, which was also done with the cut-generation algorithm, but limiting the investment budget (α) to the total investment of the corresponding solution obtained by the independent decisions strategy. This strategy evaluates the performance of our method when limited to invest at most the same amount of budget that would be used if optimal independent decisions were taken for each project. We call this a min-max limited strategy.

For this set of experiments, all problems of each solution strategy were solved to optimality within the adopted $TimeLimit$ of one hour. In Table 5.6, we summarize the results of the experiments. The rows of the table refer to the experiments for a particular uncertainty budget level and topology, while the columns are divided into strategy and the chosen metrics to be evaluated. Therefore, each cell of the table summarizes the results across all 100 different portfolios associated with 100 instances with the same topology and uncertainty budget. The idea is to compare the values of the solutions obtained by the tested independent decisions and min-max limited strategies with respect to the optimal integrated solutions obtained by the min-max strategy. Given x^* and y^* as decisions of the optimal trade-off integrated solution obtained by min-max strategy and x and y decisions of a feasible solution, we refer to the relative objective difference between those solutions as Gap, which is determined by the following function:

$$Gap(x^*, y^*, x, y) = \frac{Obj(x, y) - Obj(x^*, y^*)}{Obj(x^*, y^*)} \tag{5-47}$$

In Table 5.6, columns are defined as follows:

- **UBL**: uncertainty budget level;
- **Top.**: topology;
- **Avg. Total ET (s)**: average of total execution times for corresponding experiments;
- **Avg. Gap**: average of Gap values for corresponding experiments; and
- **Max. Gap**: maximum among Gap values for corresponding experiments.

UBL	**Top.**	**Min-Max**	**Indep. Dec.**			**Min-Max Lim.**		
		Avg. Total ET (s)	Avg. Total ET (s)	Avg. Gap (%)	Max. Gap (%)	Avg. Total ET (s)	Avg. Gap (%)	Max. Gap (%)
40%	FP	56.65	0.65	8.12	30.25	23.37	2.47	10.10
	HP	59.33	0.67	16.81	49.53	17.86	7.07	36.79
	DD	64.56	0.71	34.09	91.47	14.52	13.52	54.85
	SQ	110.22	0.73	42.16	91.40	16.35	20.49	76.51
60%	FP	105.67	0.74	5.09	25.67	88.73	0.44	2.89
	HP	116.45	0.75	11.15	39.59	92.85	2.86	22.12
	DD	128.38	0.76	21.68	79.34	93.65	5.42	31.55
	SQ	195.01	0.80	26.92	80.81	117.76	8.83	42.12
80%	FP	135.93	0.81	2.40	13.37	111.56	0.50	4.14
	HP	145.25	0.82	7.44	36.39	115.08	3.13	24.62
	DD	174.82	0.82	16.10	63.36	101.47	5.85	32.47
	SQ	213.64	0.80	20.36	62.16	145.83	9.34	47.55
100%	FP	157.54	0.91	0.00	0.00	151.30	0.00	0.00
	HP	166.94	0.88	3.82	28.12	114.43	2.08	20.26
	DD	183.85	0.87	11.66	72.17	110.50	4.44	39.89
	SQ	229.14	0.87	15.22	70.98	163.39	7.56	46.70

Table 5.6: Results for all experiments by uncertainty budget level (UBLl), topology (Top.) and strategies (min-max, independent decisions and min-max limited).

For the min-max strategy, which is the most time consuming one, all average running times were under five minutes. This provides evidence of the computational feasibility of our approach for instances of equivalent or lower dimensions. Analyzing the results for the independent decisions strategy, the average and maximum gaps were large for almost all experiments. This result illustrates the importance of taking integrated resource allocation decisions in the context of risk management of multiple interdependent projects. It also indicates that even when taking optimal decisions at the project level, the effectiveness of these decisions is far from the optimal decisions at the portfolio level. The results reveal, moreover, that when topology increases its degree of interdependency, average gaps also increase. Furthermore, the maximum gaps follow a similar pattern, with DD and FP presenting similar gap values.

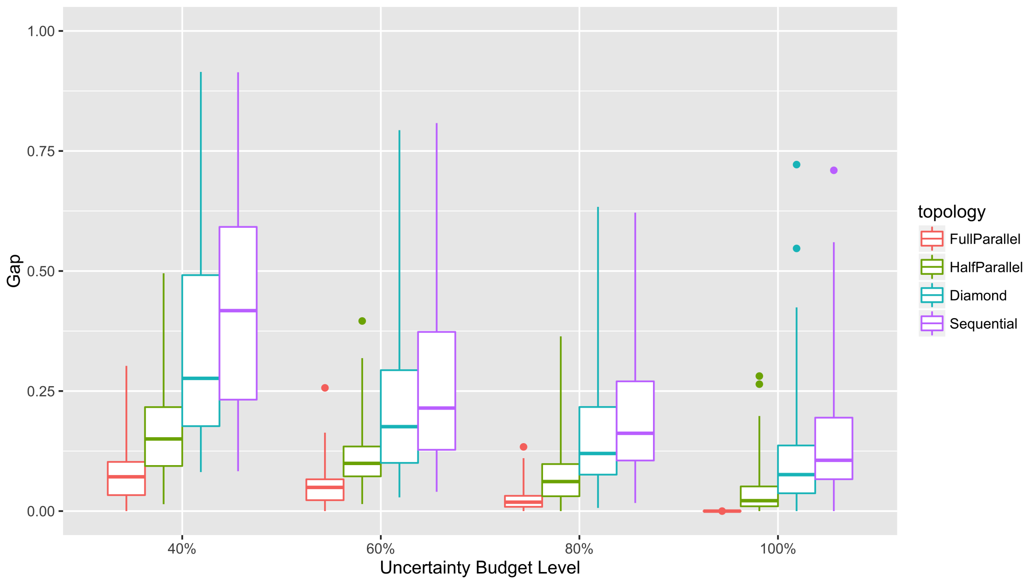


Figure 5.4: Gap x Uncertainty Budget x Topology for independent decisions strategy.

Figure 5.4 illustrates in detail how gap values are distributed by topology and uncertainty budget level. Each shape in the figure represents the gap of each of the 100 portfolios of the corresponding strategy. For a particular topology and uncertainty budget level, the boxes delimit first and third quartiles, having inside them a horizontal stroke marking the median (i.e., second quartile). The thin lines delimit minimum and maximum gap values that extend from the boxes to at most four times the interquartile range (i.e., the difference between third and first quartiles). Additionally, gap values outside this range are represented as dots and are considered outliers for our analysis. The figure supports the conclusion that as interdependency increases, gap values also increase. Therefore, the ineffectiveness of taking independent decisions at the project level, becomes more dangerous as the degree of interdependency between projects enhances. We also noticed that as the uncertainty budget level increases, gaps values tend to decrease. The reason for this pattern is depicted in the following analysis of the results of the min-max limited strategy.

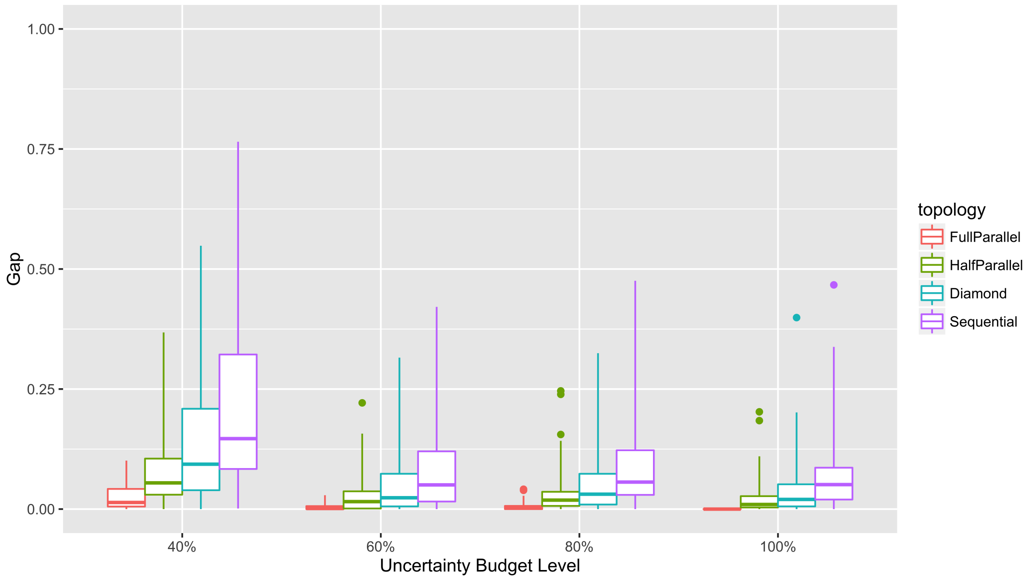


Figure 5.5: Gap x Uncertainty Budget x Topology for min-max with limited investment budget.

Figure 5.5 reflects how gap values were distributed for the min-max limited strategy. The figure depicts a great decrease in gap values when compared with the independent decisions strategy. This result is another evidence of the effectiveness of our approach, by illustrating that with the same amount of what would be invested by taking optimal decisions at the project level, the method is capable of providing much more effective resource allocation decisions. As the only difference between the analyzed strategy and the optimal strategy is the investment constraint, a gap value higher than zero means that the decisions taken independently at the project level underestimated the total amount that should be invested to achieve an optimal decision. This investment underestimation directly influences the gap results reflected in Figure 5.5; therefore, gap values at lower uncertainty budget levels are higher because the investment underestimation is also higher at these levels.

In Appendix A.1, we present detailed statistics for the experiments described in this section.

Solution Strategy Performance Assessment

For these experiments, the projects varied between 12, 16, and 20 non-dummy activities and were generated, respectively, from the `j12.mm`, `j16.mm`, and `j20.mm` instances of the MRCPSP. The number of risks generated for each project varied from 3 to 5. For each combination of number of activities and number of risks per project, we generated a set of 10 portfolios using the "Diamond" topology. Then, the resulting portfolio instances had a total number of activities equal to 48, 64, or 80 and number of risks equal to 12, 16, or 20. We also fixed β (the adversary uncertainty budget) to 60% of $\sum_{r \in R} \sigma_r^2$. The chosen fixed combination of topology and β is a standpoint from which we evaluated the proposed method varying on the dimensions of number of activities and risks. In these experiments, we also adopted an unlimited investment budget α.

In this set of experiments, the $TimeLimit$ of the cut-generation algorithm was also set to one hour. In Table 5.7, we summarize the results. Each row of the table provides the results of the 10 portfolios of a particular combination of number of activities and risks. The table columns map the chosen metrics evaluated during the experiments and are defined as follows:

- **NR**: number of risks of the portfolio;
- **NA**: number of activities of the portfolio;
- **Algorithm Gap (%)**: the relative distance in percentage between the upper (UB) and lower bound (LB) of the cut-generation algorithm (i.e.,$100 \cdot \frac{(UB-LB)}{LB}$). An algorithm gap value higher than zero means that the instance was not solved to the optimality within the imposed $TimeLimit$;
- **NNO**: number of portfolio instances that were not solved to optimality;
- **Total ET (s)**: total execution time of the cut-generation algorithm in seconds;
- **Master ET (s)**: portion of the total execution time dedicated to solving the master problem; and
- **Separation ET (s)**: portion of the total execution time dedicated to solving the separation (or adversary) problem.

NR	NA	NNO	Algorithm Gap (%)			Total ET (s)			Master ET (s)			Separation ET (s)		
			Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.
12	48	0	0.00	0.00	0.00	10.39	616.11	222.94	8.20	596.04	213.73	1.13	11.94	6.10
	64	0	0.00	0.00	0.00	52.39	464.88	198.31	47.41	455.20	188.47	3.25	11.34	6.26
	80	1	0.00	0.86	0.09	69.38	3625.80	576.37	62.55	3601.79	564.61	3.21	23.90	7.98
16	48	0	0.00	0.00	0.00	153.10	3282.32	1055.09	148.72	3263.85	1036.80	4.32	18.38	11.20
	64	2	0.00	1.89	0.26	77.98	3700.90	1746.75	75.48	3635.07	1709.64	2.44	65.73	18.91
	80	3	0.00	2.52	0.54	240.07	3691.69	2493.75	225.72	3674.71	2471.49	9.25	32.16	17.18
20	48	7	0.00	3.91	1.56	327.48	3666.98	2910.89	308.12	3657.71	2891.42	4.17	22.67	14.59
	64	9	0.00	4.23	1.54	3159.94	3670.81	3593.94	3132.72	3653.90	3571.33	12.61	42.48	22.52
	80	9	0.00	9.23	4.06	487.59	3756.81	3362.60	462.70	3740.52	3345.81	7.27	23.31	14.93

Table 5.7: Results for experiments varying number of activities and number of risks.

Analyzing the results for the experiments with at most 16 risks, the method solved to optimality 54 out of 60 instances, and the maximum algorithm gap was only 2.52%. This provides another evidence of the computational feasibility of the approach. For the experiments with 20 risks, we noticed similar good performances for 48 and 64 activities, with low average algorithm gaps, whereas the performance deteriorated for 80 activities. Overall, the method was sensitive to both number of activities and number of risks, but the influence of the number of risks appeared to be stronger. This influence was clearly observed when analyzing the increase on the total number of instances not solved to optimality when the number of risks was increased, that is, for the experiments with 12, 16, and 20 risks, we had, respectively, a total of 1, 5, and 25 instances that were not solved to optimality. For the experiments with 48, 64, and 80 activities, we had, respectively, a total of 7, 11, and 13 instances not solved to optimality. Analyzing the execution times, we noticed that solving the master problem completely dominated the total execution time of the cut-generation algorithm, with the time spent on solving the adversary problem only occupying a small portion of the total. These results provide directions for future research on improvements of the solution approach. Focus on strengthening the master problem formulation and exploring decomposition techniques are examples of promising research directions. For detailed statistics on the experiments of this section, refer to Appendix A.2.

The models and algorithms developed in this chapter tackled a complex problem that handles several decision dimensions in the context of resource allocation for project (or portfolio) management. The main idea of approaching this problem was to develop a general and flexible approach that could be easily adapted to handle different decision-making settings. For instance, one could adapt the model to minimize the worst-case sum of project durations subject to the investment budget constraint. Another adaptation could be implementing a different model for the uncertainty set that is more suitable to the decision context. For example, in qualitative risk analysis, risks are classified into probability categories (e.g., low, medium and high), a possible uncertainty set modeling approach would be to limit the number of risks that could be materialized (by the adversary) at each risk category. These two extensions are presented in Appendix B.

6
Robust Optimization Based Methodology for Project Portfolios Planning and Control

In this chapter, we explore how to combine the developed techniques in a global portfolio planning and control methodology. The main objective is to propose a method that supports the forecast, prevention, and correction of negative uncertainty effects in project objectives. The dynamic nature of the complex environment, which most projects are subject to, is the main factor driving the design of the referred methodology. In Section 6.1, we present the portfolio planning and control method itself, and Section 6.2 is dedicated to a case study of the methodology applied to a hypothetical case of the construction of two refineries, which were adapted from the example found in Hulett (2009).

6.1
Methodology

In the development of the methodology, we refer as a project to a subset of activities which execution is managed by a particular entity. In other words, it defines a decision-making frontier in the execution's operational level. Moreover, a portfolio is simply a set of interdependent projects. This interdependency may arise from various reasons such as the sharing of resources or activity precedence constraints. The portfolio is a higher-level decision instance often connected with companies' strategic objectives. The complexity involved in the execution of the work required to achieve these objectives drives companies to assign projects to different staff, divisions, or even companies. For instance, as in the example of Section 5.1.4, an EPC (Engineering, Procurement and Construction) company could create a division that is responsible for managing the environmental licensing process across all its portfolio of construction projects. In this division, detailed schedules (or sub-projects) of the licensing processes are managed separately from their corresponding construction projects. Despite this specialized division being effective in the execution of its activities, it is fundamental that the company properly distributes its resources in a way that maximizes the chance of success of the entire portfolio. Integrated resource allocation decisions are fundamental to effectively plan and control the portfolio's execution (see Chapter 5). These integrated decisions are also key aspects of the proposed methodology.

A common strategy of tracking the progress of a project is to establish periodic control points, namely work progress status (WPS, see Muriana & Vizzini (2017)). At each control point, the staff managing the project collects the progress of each activity, changing the amount of remaining work to be performed and, as a consequence, the project scheduling itself. In the context of a portfolio, our methodology proposes to have synchronized control points for all projects that compose the portfolio. At each of them, the WPS of each project is collected and the integrated resource allocation optimization model is created and solved to establish a new execution plan.

The effectiveness of the resource allocation model is directly connected to the quality of mapping and quantification of the uncertainty (i.e., risks and corresponding impacts) and the subset of actions that could be implemented to minimize these impacts (i.e., different execution modes for activities and risk mitigation-plans). Due to the dynamic nature of the referred complex portfolios, the prior mapping of all possible risks and resource allocation actions is impractical. Most practical risk management techniques propose ways to prioritize risks and possible actions (see Hulett (2009)). In our methodology, we proposed a process, at each control point, to map risks and actions associated with the most critical activities in terms of the new robust criterion described in Chapter 4. This process is used to dynamically define the relevant input parameters for the integrated resource allocation model. Furthermore, the robust criticality analysis only depends on nominal and worst-case duration estimates for each activity, which we qualified as a raw (low-level) quantification of the uncertainty. The proposed mapping of risks and corresponding impacts on activities is qualified as a high-level estimation of the uncertainty, as it is used at the portfolio level.

The methodology was also divided into the following two phases: planning and execution. Both phases have processes occurring at the project or portfolio levels. In the planning phase, the objective is to devise the initial execution plan, which may define deadlines and resource constraints. In the execution phase, the aim is to continuously distribute the available resources to effectively control the execution of the portfolio. Given these different objectives, the constructed integrated models for each phase may also differ. Despite this difference, the models and techniques described in Chapter 5 could be adapted to implement these different objectives. In the case study presented in Section 6.2, we applied one of these extensions. The following section elucidates the workflow of the methodology.

6.1.1 Workflow

Figure 6.1 presents the workflow of the proposed methodology. It contains rectangles, which represent processes, and arrows that connect inputs and outputs between them. Next, we provide descriptions of each one of these processes.

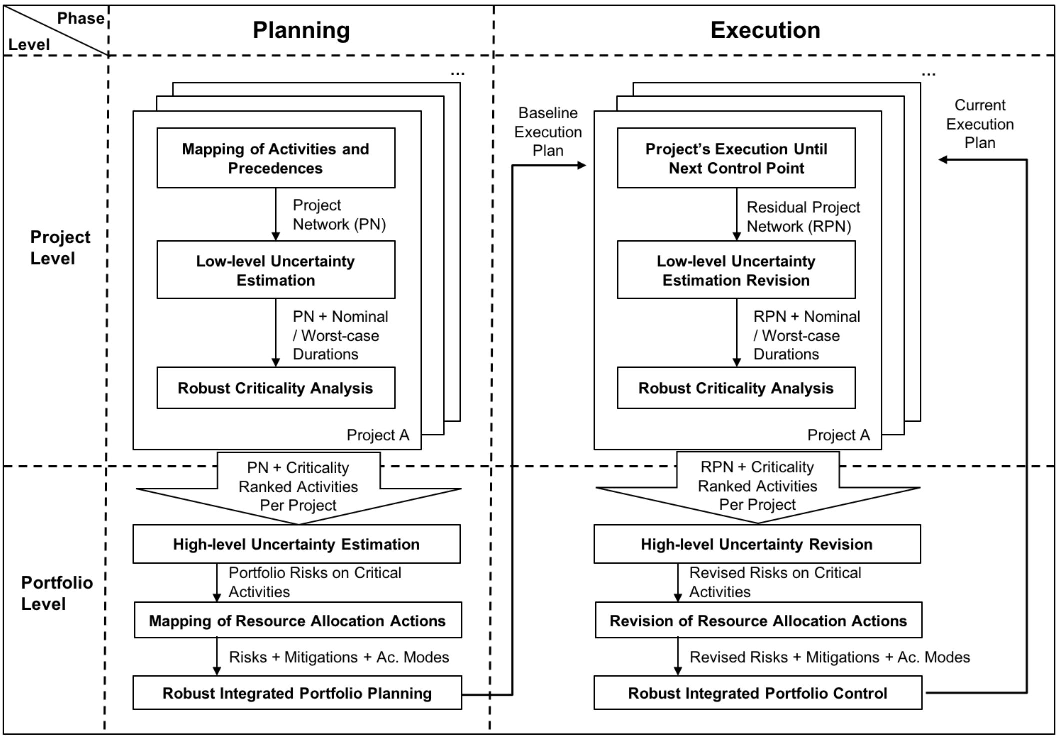


Figure 6.1: Portfolio's Planning and Control Robust Methodology.

The planning processes that should run for each project are:

– **Mapping of activities and precedences**: define the set of activities that should be performed in the context of a project and design the precedence relations between them. This process generates the project network (PN);

– **Low-level uncertainty estimation**: estimate nominal and worst-case durations for each activity. These estimations could be obtained by the analysis of historical data or interviewing activity-related specialists as described in Hulett (2009). They reflect a raw and simple quantification of the uncertainty affecting the project's execution and is the basis of the robust criticality analysis process; and

– **Robust criticality analysis**: solve the robust criticality model, proposed in Chapter 4, for different values of α and β in the mapped PN with corresponding duration estimates. As demonstrated, traditional critical path analysis fails to capture worst-case disruptions of the uncertainty, which justifies the use of the robust criterion. The results provide project duration estimations under different uncertainty conditions (see duration heat map in Section 4.3) and also rank activities in terms of robust criticality (see mitigation frequency in Section 4.3). This ranking of activities supports decision makers to decide which activities require focus to effectively control the project's execution.

The planning phase at the portfolio level start by collecting all projects' networks and corresponding criticality analysis results. With this data, the following processes are executed:

– **High-level uncertainty estimation**: map the risks affecting the critical activities in terms of the robust criterion. Some risks may affect multiple activities at the same or different projects. Corresponding impacts should also be estimated. This estimation could also be obtained by historical data analysis or interviews with domain specialists. In the risk management practice, this process is often referred as risk register (see Hulett (2009)). In this work, it has the role of mapping and quantifying the uncertainty environment at the portfolio level, which is used as input for the integrated robust optimization model;

– **Mapping of resource allocation actions**: given mapped risks and ranked critical activities, this process is responsible for mapping resource allocation actions that could be performed to minimize uncertainty impacts. At the activity level, one could map different execution modes that may decrease corresponding durations. At the risk level, mitigation plans could be designed to potentially alleviate corresponding impacts. Each of these actions have their own resource requirements; and

– **Robust integrated portfolio planning**: precedence relations between activities at different projects are established to construct an integrated portfolio network. Considering the objective of the planning phase, resource constraints, mapped risks and actions, apply the techniques developed in Chapter 5 to construct and solve the resource allocation planning model, over the integrated portfolio network, determining the best combination of mitigation actions and activity modes, minimizing the worst-case uncertainty impacts on the portfolio's objective. The selected mitigation actions are implemented and the resources are distributed across projects in line with the corresponding activity execution modes. To reflect these decisions, a new scheduling plan is defined for each project, which may also establish their deadlines. We call the final output of this process the baseline execution plan.

The execution phase is triggered after the creation or revision of the execution plan. As in the planning phase, the initial processes are executed at the project level, we define them as follows:

- **Project's execution until next control point**: following the defined scheduling and modes of activities, which are specified in the execution plan, the project is executed until the next control point. Short-term decisions during this period could be driven by a new robust criticality analysis. At the end of the control point, the progress of the activities should be collected, reflecting the project's execution performance and the materialization of the uncertainty in the referred period. The result of this process is a residual project network (RPN), which represents the remaining work to be performed;
- **Low-level uncertainty estimation revision**: given the progress of the activities and the uncertainty realization, new estimates for remaining nominal and worst-case durations should be defined; and
- **Robust criticality analysis**: this is the same process as defined in the planning phase, but with the analysis being performed in the RPN with revised estimates for activity durations and worst-case uncertainty impacts.

The execution phase at the portfolio level also starts by collecting all residual projects' networks and corresponding criticality analysis results. The processes that follow are similar to the ones of the planning phase, they are defined as follows:

- **High-level uncertainty revision**: given the dynamic nature of the execution of complex portfolios, a new risk-based uncertainty quantification should be performed considering the RPN and the new ranking of criticality for the activities. This process follows the same methodology previously described in the planning phase;
- **Revision of resource allocation actions**: the resource allocation actions should also be revised to reflect the new quantification of the uncertainty and the current progress of activities. This process also follows the same methodology of the corresponding process of the planning phase; and

– **Robust integrated portfolio control**: in this process, the integrated portfolio network is assembled from the RPNs and corresponding interdependencies. With this integrated network and the current mapping of risks and resource allocation actions, the robust optimization model, developed in Chapter 5, is constructed and solved to obtain an effective resource allocation on activities modes and mitigation plans, that minimizes the portfolio's total cost of the worst-case uncertainty set scenario. As in the planning phase, the selected mitigation actions are implemented and the resources are distributed across projects according with the corresponding activity execution modes. The scheduling of each project is revised to reflect the new execution plan. The methodology processes of the execution phase are then re-started to implement this current plan.

In the next section, we provide an example of the application of this proposed methodology.

6.2 Case Study

The case study described in this section was based on a simplified refinery construction project extracted from Hulett (2009). We exercised the methodology on a portfolio for the construction of two twin refineries, hypothesizing the case where each one would be installed in different regions of the same construction site. Then, two projects were created, namely north refinery construction and south refinery construction. These projects share the same resources and are interdependent, as we detail later. The set of activities to be performed at each project was also the same. Next, we discuss each phase of the methodology until the robust integrated portfolio control process for the first execution control point.

Activity, a	d_a^1	$d_a^1 + \Delta_a$	*Predecessors*
Preliminary Authorization - North, *PA-N*	30	42	
FEED Design - North, *FD-N*	350	489	*PA-N*
Authorization - North, *A-N*	30	42	*FD-N*
Balance of Procurement - North, *BP-N*	650	909	*A-N*
Vendor Data Available - North, *VDA-N*	500	700	*A-N*
Detailed Design - North, *DD-N*	570	798	*A-N*
Procurement of LLE - North, *PL-N*	775	1085	*FD-N*
Site Work - North, *SW-N*	100	140	*A-N*
Construction Before LLE - North, *CBL-N*	600	840	*SW-N*
Construction After LLE - North, *CAL-N*	230	322	*CBL-N, PL-N*
Commissioning - North, *C-N*	110	154	*CAL-N*

Table 6.1: Activities and Precedences for North Refinery Construction.

The first processes of the planning phase are executed at the project level. Given the equivalence of both portfolio projects, at this stage, it is sufficient to analyze only one of them. In Table 6.1, we provide the result of the first two planning processes: mapping of activities and precedences and low-level uncertainty estimation. The table specifies for each activity of the north refinery project: name, code, nominal duration (d_a^1), worst-case duration ($d_a^1 + \Delta_a$), and predecessors. The activities also have name and code suffixes to specify which project they belong to, which is a necessary differentiation when analyzing results in the integrated portfolio network.

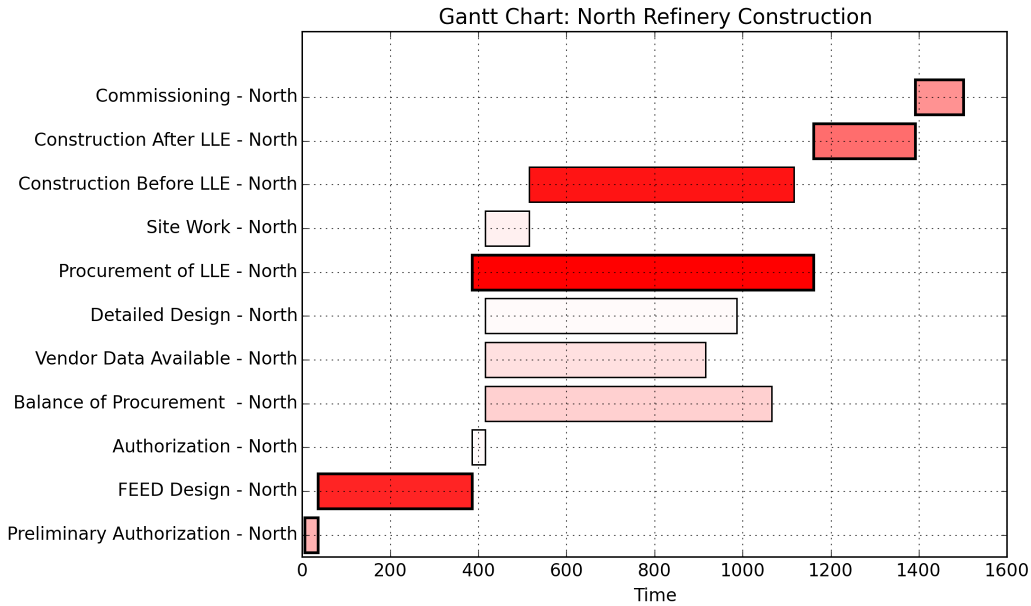


Figure 6.2: Robust Criticality Analysis Gantt Chart for North Refinery Construction.

The next process of the methodology was to perform a robust criticality analysis on the mapped PN with the corresponding duration estimates. In Figure 6.2, we provide the Gantt Chart mitigation frequency heat map for the north refinery project. As detailed in Chapter 4, to build this graph, several models were solved for different values of α (i.e., number of activities to have uncertainty mitigated) and β (i.e., maximum number of activities assuming their worst-case durations) parameters. The redness of each activity is proportional to the number of times it was chosen as part of the optimal mitigation solution. This indicate which activities are the most essential and on which one should focus the effort of minimizing the impact of the uncertainty. In this case, with respect to the robust criterion, the following five activities were clearly more critical: front-end engineering design (FEED) (FD), procurement of long lead equipment (LLE) (PL), construction before LLE (CBL), construction after LLE (CAL), and Commissioning (C). The FEED phase focuses on technical requirements and basic engineering design. PL, CBL, CAL are activities involved with the process of acquisition and installation of LLE such as heat exchangers and heavy-walled vessels. Commissioning is the process of testing and validating the refinery equipments and processes before starting client operations. The result of this analysis was used as input for the next process of the planning phase. Notice that the activity CBL is not part of the critical path, as we have demonstrated in Chapter 4, traditional critical path analysis is not always effective to select the best activities to mitigate uncertainty impacts in the advent of simultaneous worst-case realizations. This process of filtering activities to focusing on would be more relevant in huge projects, where it is almost impossible to map all risks and corresponding impacts for every activity.

Risk, r	*Prob.*	σ_r^1	*List of r Impacts in Duration or Cost* $= \{(a, \Delta_{ar}^1 = \theta_{ar}^1), \forall a \in I_r\}$
Design Productivity may be lower than expected, *R1*	0.9	10	(*FD-N*, 85), (*DD-N*, 114), (*FD-S*, 85), (*DD-S*, 114)
Construction logistics may be harder than expected, *R2*	0.9	10	(*SW-N*, 15), (*CBL-N*, 90), (*CAL-N*, 34.5), (*SW-S*, 15), (*CBL-S*, 90), (*CAL-S*, 34.5)
Construction supervision may be scarce, *R3*	0.5	50	(*SW-N*, 25), (*CBL-N*, 150), (*CAL-N*, 57.5), (*SW-S*, 25), (*CBL-S*, 150), (*CAL-S*, 57.5)
Vendor reps may be scarce for commissioning, *R4*	0.35	65	(*C-N*, 44), (*C-S*, 44)
LLE suppliers may be busy, *R5*	0.7	30	(*PL-N*, 180), (*PL-S*, 180)

Table 6.2: Portfolio Risks in the Planning Phase.

At the portfolio level, the first process of the planning phase is to map and quantify the uncertainty affecting the most critical activities of all projects. As mentioned, this can be done by analyzing historical data on similar projects or through interviews with specialists. In our case, the process was performed for the five previously selected activities and its results are presented in Table 6.2. Five main risks were identified, and, for each one of them, a probability of materialization and the set of affected activities with corresponding impact magnitudes in duration and cost were estimated. The probability estimation was used as a proxy for establishing the values of the uncertainty budget requirement parameters σ_r. In this case, we applied a simple inversely proportional function $\sigma_r = (1.0 - Probability_r) * 100$. Other functions may be used to implement the property that as chances of materialization increase, σ_r values should decrease (see Chapter 5). For ease of presentation, we assumed that impacts in duration were of the same magnitude as the ones in cost (i.e., $\Delta^1_{ar} = \theta^1_{ar}$). *R1* is a risk connected with design activities (*FD* and *DD*), while *R2* and *R3* are associated with the construction ones (*SW*, *CBL* and *CAL*). These risks affect activities of both north and south projects, which is the same for *R4* and *R5*. This highlights the fact that this process is performed at the portfolio level. *R4* and *R5* are related to commissioning (*C*) and procurement of LLE (*PL*) activities.

r	*Mitigation*	h^2_r	*Prob. After Mit.*	σ^2_r	*List of r Impacts in Duration or Cost After Mit.* $= \{(a, \Delta^2_{ar} = \theta^2_{ar}), \forall a \in I_r\}$
R1	Extra investment to guarantee dedicated high-skilled engineers	588	0.35	65	(*FD-N*, 35), (*DD-N*, 57), (*FD-S*, 35), (*DD-S*, 57)
R2	Contract a benchmark logistics company	446.4	0.1	90	(*SW-N*, 5), (*CBL-N*, 30), (*CAL-N*, 11.5), (*SW-S*, 5), (*CBL-S*, 30), (*CAL-S*, 11.5)
R3	Extra investment to guarantee dedicated high-skilled supervisors	139.5	0.2	80	(*SW-N*, 15), (*CBL-N*, 90), (*CAL-N*, 34.5), (*SW-S*, 15), (*CBL-S*, 90), (*CAL-S*, 34.5)
R4	Allocate extra budget for vendors exclusivity agreements	26.4	0.2	80	(*C-N*, 11), (*C-S*, 11)

Table 6.3: Mitigation Actions for Portfolio's Planning.

Activity, a	d^1_a	c^1_a	d^2_a	c^2_a
FEED Design - North, *FD-N*	350	350	270	590
Procurement of LLE - North, *PL-N*	775	775	650	1108
Construction Before LLE - North, *CBL-N*	600	600	540	780
Construction After LLE - North, *CAL-N*	230	230	200	320
FEED Design - South, *FD-S*	350	350	270	590
Procurement of LLE - South, *PL-S*	775	775	650	1108
Construction Before LLE - South, *CBL-S*	600	600	540	780
Construction After LLE - South, *CAL-S*	230	230	200	320

Table 6.4: Crashing Actions for Portfolio's Planning.

After the risks mapping comes the process of designing the resource allocation actions (mitigation plans and activity crashing modes) that could minimize or absorb uncertainty impacts. A mitigation plan is an action that could simultaneously alleviate the multiple impacts of a particular risk, while a crashing action is an extra resource allocation for the execution of an activity to potentially decrease its duration. In Table 6.3, we present the designed mitigation actions; the corresponding mitigated risk (r), an implementation cost (h_r^2), probability, uncertainty budget requirement parameter (σ_r^2) and risk impacts in the case of mitigation implementation are specified for each one. Regarding the execution modes, we considered the baseline modes for the activities to have equal durations and costs (i.e., $d_a^1 = c_a^1$). In Table 6.4, we provide the activities that could have their durations decreased to d_a^2 by extra resource allocation resulting in a cost of c_a^2, which we referred to as crashing modes. In our case, they reflect the action of increasing the number of workers performing the corresponding activity, that is, extra engineers (for *FD-N* or *FD-S*), procurement specialists (for *PL-N* or *PL-S*), or construction workers (for *CBL-N*, *CBL-S*, *CAL-N*, or *CAL-S*).

Given the mapping of risks, mitigations, and crashing actions, the next process of the methodology is build and solve the robust integrated portfolio planning model. First, precedence relations between activities of different projects are identified to create the integrated portfolio network. In our case, the company identified that commissioning activities of both projects could not be executed simultaneously, due to the highly specialized and limited team required to execute them. Then, managers decided that commissioning at the north refinery would be executed prior to the one at the south, creating a precedence relation with *C-N* as predecessor and *C-S* as successor.

At this planning phase, the company's objective was to devise a portfolio execution plan that is robust against uncertainty disruptions and that, simultaneously, minimizes the durations of both projects. This initial plan defines the deadline commitments with clients to start refineries' operations. Not attending these deadlines will result in penalty charges for the company, so the robustness of the plan is a critical factor. It is also important to start operations as soon as possible, which motivates the objective of minimizing project durations. Within this context, we adapted the portfolio optimization model, presented in Chapter 5, to determine the best combination of activity modes and risk-mitigation actions that minimizes the sum of project durations of the worst-case uncertainty set scenario subject to the investment budget constraint. This adaptation is detailed in Appendix B. The adversary uncertainty budget was set to $\beta = 82.5$, which was half of the sum $\sum_{r \in R} \sigma_r^1$. Given that the baseline cost is $\sum_{a \in A} c_a^1 = 7890$, we set the investment budget one thousand monetary units higher than the baseline, that is, $\alpha = 8890$. The model was then solved with these parameters.

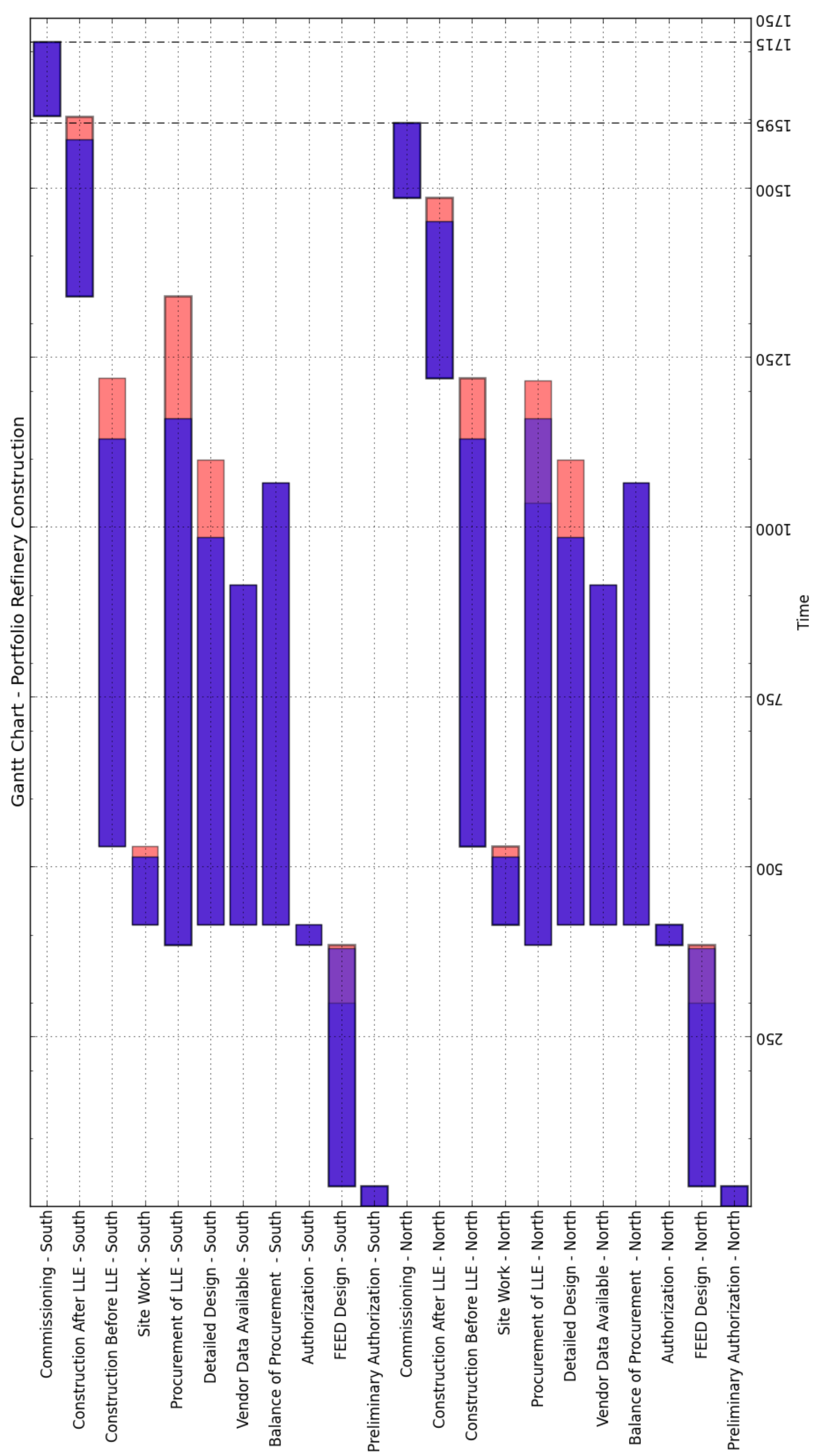


Figure 6.3: Gantt Chart of Portfolio's Planning Solution.

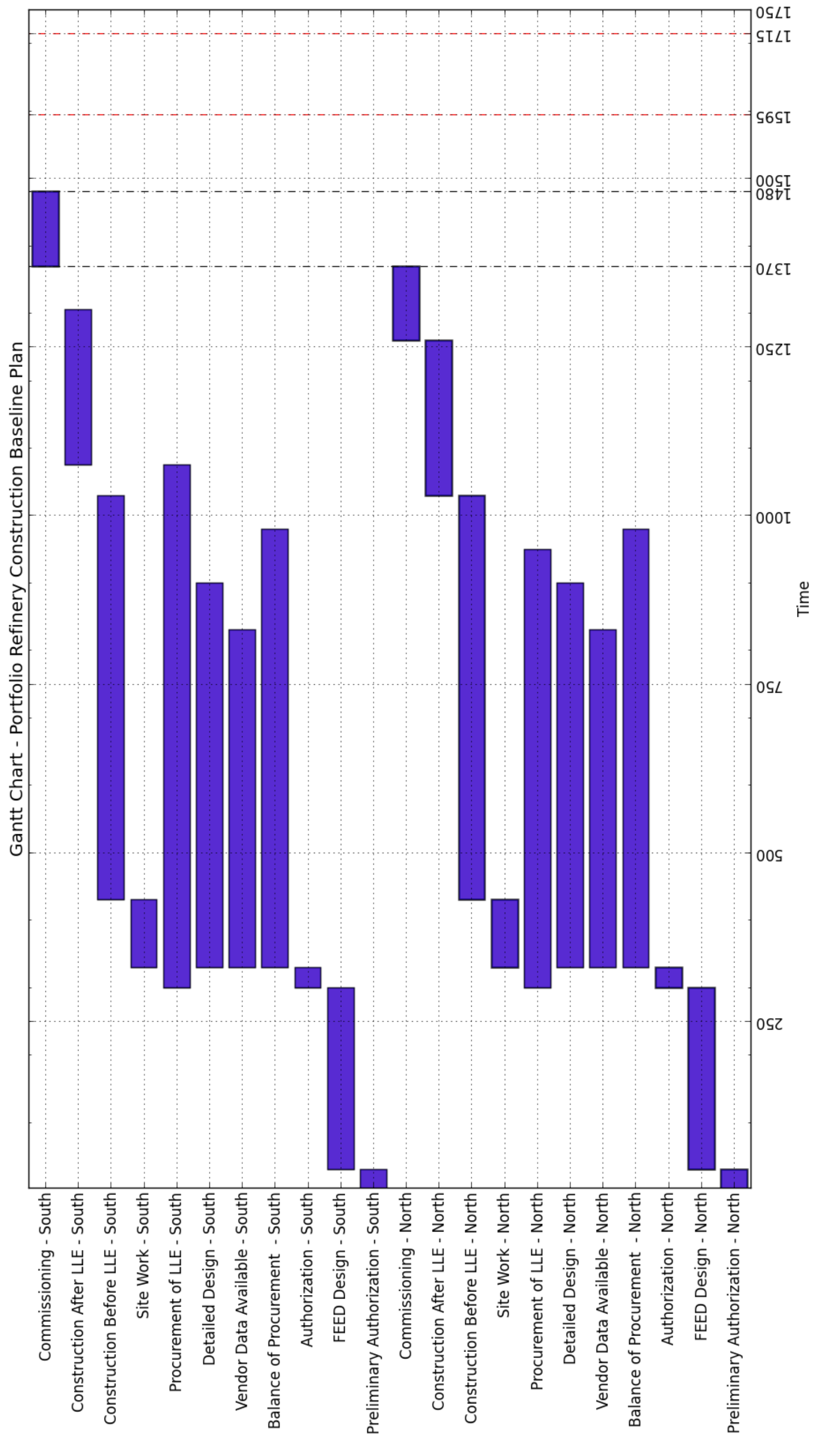


Figure 6.4: Gantt Chart of Portfolio's Baseline Execution Plan.

The optimal obtained resource allocation solution involved implementing the mitigation plan of risk *R1* (extra investment to guarantee dedicated high-skilled engineers), execute activities *FD-N*, *FD-S*, and *PL-N* on their crashing modes, and the others on their baseline modes. In Figure 6.3, we present the scheduling of the portfolio activities for the optimal planning solution in the worst-case uncertainty scenario. The projects' deadlines were set according to this schedule, providing the necessary protection against the uncertainty. The north refinery is finishing at time unit (TU) 1595, while the one at the south is finishing at TU 1715. The red part of the activity bars represents the risk impacts of the worst-case scenario. In this case, the materialization of risks *R1*, *R2*, and *R5*. The blue part of the bars denotes activity durations at the selected modes, while purple regions signify the difference between baseline and crashing durations. From the analysis of this schedule, we notice that crashing activity "Procurement of LLE - North" ($PL\text{-}N$) prevents both projects to finish later, given the precedence relation between the final commissioning activities. This is a clear evidence of the importance of taking integrated resource allocation decisions for portfolios of interdependent projects.

The final objective of the planning phase is to devise the baseline execution plan considering the implementation of the resource allocation decisions of the optimal solution. Figure 6.4 depicts the baseline plan for our case study. The vertical red lines indicate the deadlines of each project, while the black ones mark corresponding planned finishes. The differences between these references are the protection against the uncertainty to which we referred earlier. The next step was then to follow the devised plan and execute projects until the next control point, starting the execution phase of the methodology.

Activity, a	*Progr.*	d_a	*Start*	*Finish*	*Rem. Dur.*
Preliminary Authorization - North, *PA-N*	1.00	30	0	30	0
FEED Design - North, *FD-N*	1.00	270	30	360	0
Authorization - North, *A-N*	1.00	30	360	390	0
Balance of Procurement - North, *BP-N*	0.17	650	390		540
Vendor Data Available - North, *VDA-N*	0.22	500	390		390
Detailed Design - North, *DD-N*	0.19	570	390		460
Procurement of LLE - North, *PL-N*	0.22	650	358		508
Site Work - North, *SW-N*	0.00	100			100
Construction Before LLE - North, *CBL-N*	0.00	600			600
Construction After LLE - North, *CAL-N*	0.00	230			230
Commissioning - North, *C-N*	0.00	110			110
Preliminary Authorization - South, *PA-S*	1.00	30	0	30	0
FEED Design - South, *FD-S*	1.00	270	30	360	0
Authorization - South, *A-S*	1.00	60	360	420	0

Activity, a	*Progr.*	d_a	*Start*	*Finish*	*Rem. Dur.*
Balance of Procurement - South, *BP-S*	0.12	650	420		570
Vendor Data Available - South, *VDA-S*	0.16	500	420		420
Detailed Design - South, *DD-S*	0.14	570	420		490
Procurement of LLE - South, *PL-S*	0.18	775	358		633
Site Work - South, *SW-S*	0.00	100			100
Construction Before LLE - South, *CBL-S*	0.00	600			600
Construction After LLE - South, *CAL-S*	0.00	230			230
Commissioning - South, *C-S*	0.00	110			110

Table 6.5: Progress for Portfolio's Activities at First Control Point.

At the end of a control point, the progress of each activity is measured, reflecting the materialization of the uncertainty in the period. The output of the first process of the execution phase is a progress report, which results in revised networks for each project mapping the remaining work to be performed. In Table 6.5, we present the progress report of portfolio activities for our case study after the first control point at TU 500. For each activity, its current progress, the planned total duration (d_a), the actual start and finish instants, and remaining duration were specified. Figure 6.5 depicts the revised scheduling mapping the current progress. The green part of activity bars represents the work performed. By analyzing this schedule, we noticed that risk *R1* materialized and affected design activities. Hence, although the portfolio was delayed from its baseline plan, both projects still have a slack for their respective deadlines.

The next processes of the methodology, prior to the "robust integrated portfolio control", are dedicated to revise the uncertainty mapping and resource allocation actions. We excluded their details, since they are similar to corresponding processes at the planning phase. The revision results are demonstrated in Tables 6.6, 6.7, and 6.8. As we can be observed, the set of risks have changed; *R1* was materialized, while *R2* and *R3* were discarded. A new unpredicted risk also appeared (*R6*), mapping the threat of strike of construction workers due to a bad working environment at the site. Moreover, the employees' demands were mapped in the mitigation action to improve the work infrastructure.

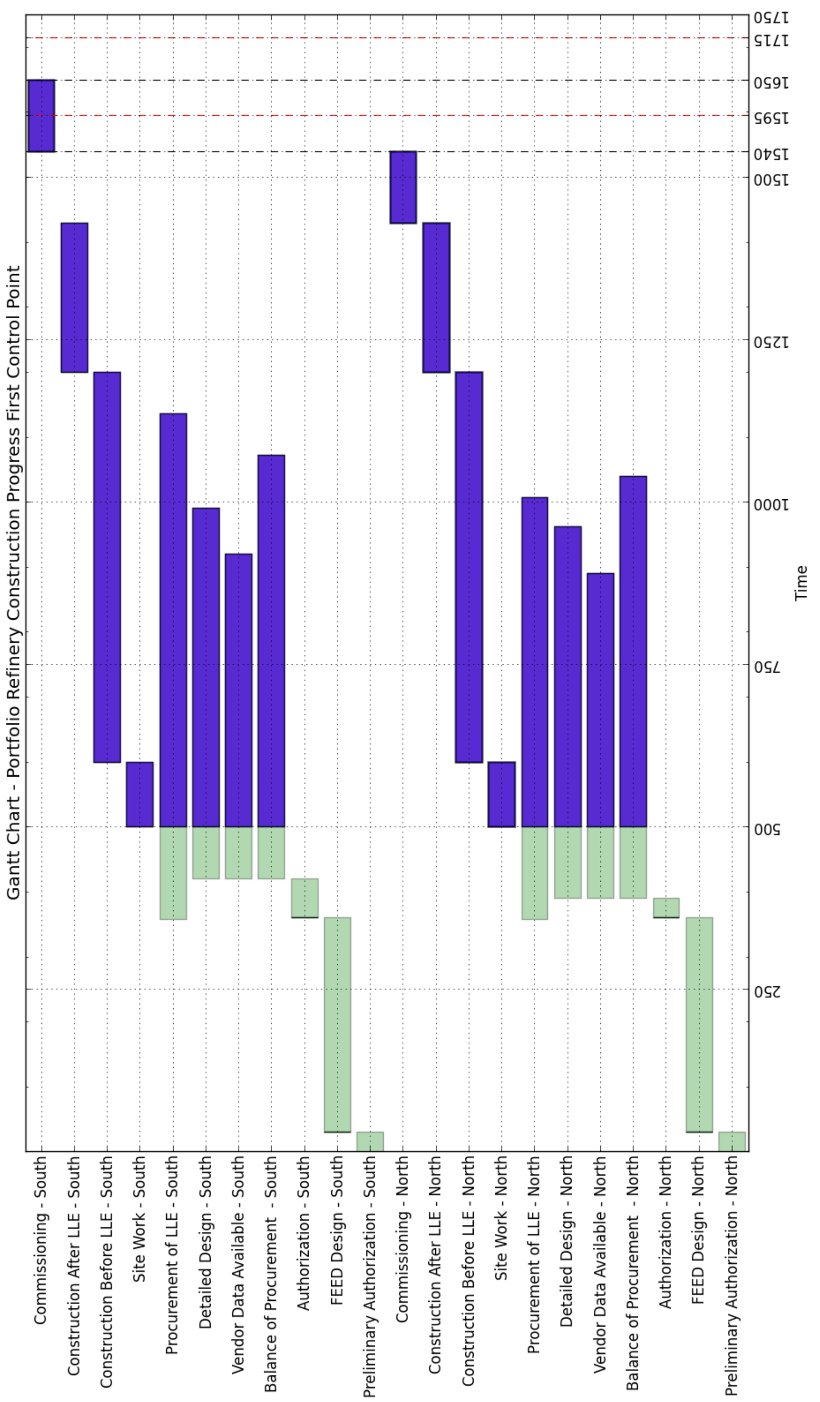


Figure 6.5: Gantt Chart of Progress for Portfolio's First Control Point.

Risk, r	*Prob.*	σ_r^1	*List of r Impacts in Duration or Cost* $= \{(a, \Delta_{ar}^1 = \theta_{ar}^1), \forall a \in I_r\}$
Vendor reps may be scarce for commissioning, *R4*	0.35	65	(*C-N*, 44), (*C-S*, 44)
LLE suppliers may be busy, *R5*	0.7	30	(*PL-N*, 180), (*PL-S*, 180)
Strike of construction workers, *R6*	0.9	10	(*SW-N*, 60), (*CBL-N*, 60), (*CAL-N*, 60), (*SW-S*, 60), (*CBL-S*, 60), (*CAL-S*, 60)

Table 6.6: Portfolio Risks in the Execution Phase First Control Point.

r	*Mitigation*	h_r^2	*Prob. After Mit.*	σ_r^2	*List of r Impacts in Duration or Cost After Mit.* $= \{(a, \Delta_{ar}^2 = \theta_{ar}^2), \forall a \in I_r\}$
R4	Allocate extra budget for vendors exclusivity agreements	26.4	0.2	80	(*C-N*, 11), (*C-S*, 11)
R6	Improve construction work infrastructure	200	0.1	90	(*SW-N*, 10), (*CBL-N*, 10), (*CAL-N*, 10), (*SW-S*, 10), (*CBL-S*, 10), (*CAL-S*, 10)

Table 6.7: Mitigation Actions for Portfolio's Control.

Activity, a	d_a^1	c_a^1	d_a^2	c_a^2
Procurement of LLE - South, *PL-S*	775	775	675	1078
Construction Before LLE - South, *CBL-S*	600	600	540	780
Construction After LLE - South, *CAL-S*	230	230	200	320

Table 6.8: Crashing Actions for Portfolio's Control.

At this phase, the main objective is to devise a cost-effective execution plan for the remaining work, taking into account the possible penalties for delays. This is exactly the problem modeled and solved in Chapter 5. Subsequently, we solved the referred model for the current mapped risks and resource allocation actions, adopting the same β value of the planning phase (82.5), and without the investment constraint, as we wanted to obtain the best investment-cost trade-off plan. The optimal resource allocation plan obtained involved implementing mitigation actions for risks *R4* and *R6*. This means that, at the adopted uncertainty hypothesis, it is cost-effective to improve the infrastructure at the construction site and to establish exclusivity agreements with commissioning vendors at the expense of extra fees. The schedule at the worst-case scenario of the optimal control resource allocation solution is presented in Figure 6.6.

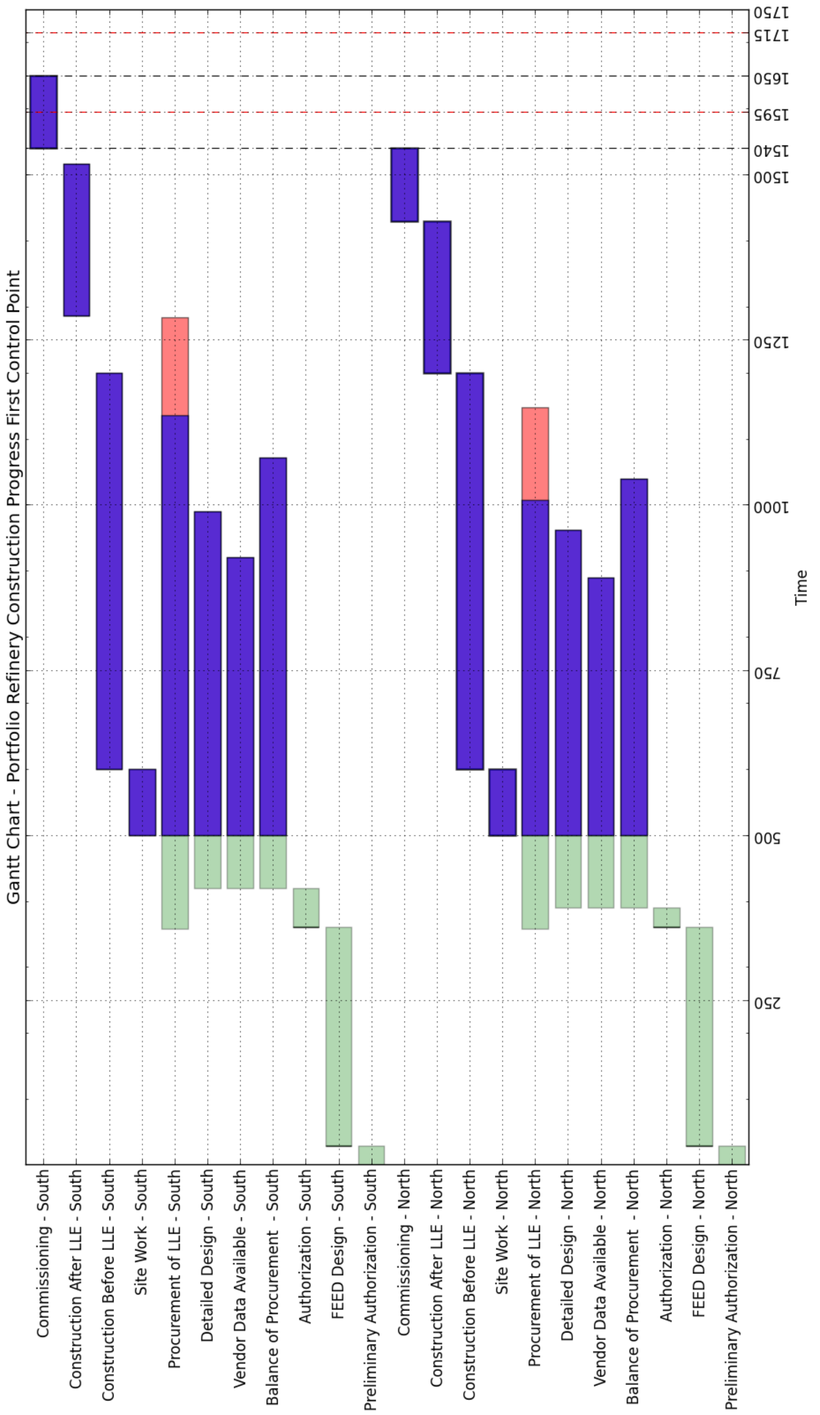


Figure 6.6: Gantt Chart of Portfolio's Control Solution.

Overall, this case study demonstrated important capabilities of the proposed planning and control portfolio methodology and the developed techniques of Chapters 4 and 5. The capacity of identifying key activities that are not part of the traditional critical path and the flexibility of the proposed robust integrated portfolio model are examples of the show-cased features of the methods developed in this work.

7
Conclusions

In this work, we have developed a robust optimization based methodology for planning and control project portfolios under uncertainty. The method combines models and algorithms for multiple resource allocation problems in project (or portfolio) management under a robust optimization perspective. The common proposed framework applied to these problems models the uncertainty environment as an adversary that selects the worst-case combination of risks for any decision maker's actions. Within this context, the main goal of the decision marker is to determine optimal resource allocation plans for minimizing a particular objective subject to the assumption that the adversary's worst-combination of risks will materialize. The approach also provides a way to control the degree of conservatism of the solutions. For each problem, a solution strategy was developed through a reformulation scheme from a compact min-max formulation to a cut-generation algorithm. Several computational experiments were conducted for each problem. These experiments highlighted important insights that driven the design of the referred portfolio planning and control methodology. These experiments also provided evidence of the effectiveness and computational feasibility of the proposed solution strategies. The application of the methodology was demonstrated in a case study of a simplified representation of a portfolio aimed at the construction of two twin refineries in different regions of the same site. This example illustrated important capabilities of the proposed methods in a practical context. In general, the work provides a flexible framework to model complex decision-making realities in the context of project scheduling and risk analysis. Next, we detail the concluding remarks for the chapters containing the main contributions.

In Chapter 4, we have developed a new robust optimization based criticality criterion that accounts for the combined effects of delays in multiple project activities. The modeled problem aims at determining the activities of a project that should be the focus of uncertainty mitigation measures – in the sense that resource and effort should be put into place so as to ensure that their actual durations equal their original nominal estimates. The proposed model also provides an upper bound on the project's total duration under the assumed worst-case hypothesis. The developed solution strategy resulted in a cut-generation procedure with a polynomial-time separation algorithm, while most criticality indexes, for projects with uncertain durations, demand Monte-Carlo simulations to be determined. The computational experiments provided evidence of the relevance for the new criterion by revealing that traditional critical path analysis fails to capture the combined worst-case effects of the uncertainty.

In Chapter 5, we have applied the proposed robust optimization approach to a cost-investment trade-off problem aimed at determining optimal activity execution modes and risk-mitigation plans for a portfolio of interdependent projects subject to multiple risks and tardiness penalties for delays. The computational experiments highlighted the importance of taking integrated resource allocation decisions in the context of uncertainty mitigation for a portfolio of interdependent projects. The degree of interdependency between the projects is a crucial factor in this context. The experiments also provided evidence of the effectiveness and computational feasibility of the approach.

In Chapter 6, we have detailed the final global portfolio planning and control methodology. We have proposed a general workflow and described its main processes and interfaces connecting them. The methodology was designed to consider the following main principles:

- complex project portfolios are inherently uncertain and dynamic, so it is impractical to map and quantify all the risks that they may be subject to in advance;
- focusing uncertainty mitigation measures only on the activities of the critical path may not be effective under simultaneous worst-case realizations of activity durations; and
- for portfolios of interdependent projects, it is crucial to devise resource allocation plans in an integrated context.

The models and algorithms proposed in Chapters 4 and 5 have formed the main building blocks for implementing the methodology's processes. The presented case study demonstrated the application of the methodology, highlighting the importance of the adopted main principles in a practical context.

7.1 Future Work and Extensions

From the perspective of the robust integrated portfolio model, a future research effort could investigate different approaches for modeling the adversary uncertainty set. For instance, in a context of qualitative risk analysis, risks are classified in probability categories (e.g., low, medium, and high), a possible uncertainty set modeling approach would be to limit the number of risks that could be materialized (by the adversary) at each risk category. The expansion of the modeled constraints, such as to incorporate renewable resource requirements as in the RCPSP model, and the study of variants in terms of the portfolio objectives, such as the minimization of the sum of project durations presented in Appendix B, are also valuable research directions. Furthermore, the robust optimization model could also be explored in different problems, which have the same characteristics found in the portfolio resource allocation problem such as the decision-dependent uncertainty aspect and the simultaneous effect of an uncertain parameter on rows and columns of the problem model.

From the perspective of the solution methods, heuristic procedures could be developed to determine initial solutions to the cut-generation algorithms. The objective value of a heuristic solution would initialize the upper bound (UB) variable, providing a closer initial gap to the lower bound (LB) and potentially decreasing the number of iterations required for convergence. Finally, focusing on strengthening the master problem formulations and exploring decomposition techniques are also suggestions for future research.

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A
Summary Statistics of Experiments for the Investment-Cost Tradeoff Portfolio Problem

A.1
Effectiveness of Integrated Decisions

A.1.1
Instance Parameters

In Table A.1, we present minimum, maximum and average values of model parameters, that are independent from the uncertainty budget levels (UBL) and topologies (Top.), of all generated instances of Section 5.3.2. We omitted c_a^1, c_a^2 and h_r^1, as described in Section 5.3.1 $c_a^1 = d_a^1$, $c_a^2 = 2 \cdot c_a^1$ and $h_r^1 = 0$. In Table A.2, we present statistics for the due date parameter τ_p that is topology dependent.

Param.	Min.	Max.	Avg.
ρ_p	1	9	4.65
d_a^1	1	10	7.94
d_a^2	1	8.0	5.51
σ_r^1	30	95	64.80
σ_r^2	51	99	84.69
Δ_{ar}^1	1	10	5.13
Δ_{ar}^2	0	7	1.89
θ_{ar}^1	1	36	10.59
θ_{ar}^2	0	25	4.33
h_r^2	2	147	25.90

Table A.1: Summary statistics of instance parameters independent from UBL and topology.

Top.	Min.	Max.	Avg.
FP	19	53	34.92
HP	19	99	52.52
DD	24	141	70.80
SQ	24	185	87.37

Table A.2: Summary Statistics of τ_p parameter for each topology (Top.).

In Table A.3, we present the statistics for the instance parameters that depend on the UBL, they are the β itself and the investment limit α that is used by the Min-Max Limited strategy, the other strategies have unlimited α.

UBL	α (Min-Max Lim.)			β		
	Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	342.0	603.0	446.73	366.8	446.0	406.508
60%	415.0	822.0	546.5	550.2	669.0	609.762
80%	405.0	745.0	541.43	733.6	892.0	813.016
100%	394.0	754.0	554.9	917.0	1115.0	1016.27

Table A.3: Summary Statistics of parameters dependent from the uncertainty budget level (UBL): α used in the tests of Min-Max Limited strategy and β.

A.1.2 Solution Statistics

In this section, we present summary solution statistics of Section's 5.3.2 experiment for each strategy, uncertainty budget level and topology. The following tables are related with the cost components of the solutions. In Table A.4, we present statistics for the investment in activities modes ($\sum_{a\in A}\sum_{m\in M_a} c_a^m x_a^m$). In Table A.5, the presented statistics refer to the investment in mitigations ($\sum_{r\in R}\sum_{l\in L_r} h_r^l y_l^r$), while in Table A.6, the presented values summarize the total investment ($\sum_{a\in A}\sum_{m\in M_a} c_a^m x_a^m + \sum_{r\in R}\sum_{l\in L_r} h_r^l y_l^r$). In Table A.7, we present statistics for the total cost impact in activities due to materialized risks ($\sum_{a\in A}\sum_{r\in R_a}\sum_{l\in L_r} \theta_{ar}^l z_r^l$), while in Table A.8, we summarize the costs due to penalties for delays ($\sum_{p\in P} \rho_p o_p$). In Table A.9, we provide statistics for the objective value of each solution, that is, the portfolio total cost ($\sum_{a\in A}\sum_{m\in M_a} c_a^m x_a^m + \sum_{r\in R}\sum_{l\in L_r} h_r^l y_l^r + \Pi(x,y)$).

UBL	Top.	Min-Max			Indep. Dec.			Min-Max Lim.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	FP	296.0	457.0	369.35	304.0	457.0	370.65	296.0	451.0	357.91
	HP	312.0	474.0	388.21	304.0	457.0	370.65	306.0	454.0	367.73
	DD	321.0	473.0	398.87	304.0	457.0	370.65	308.0	455.0	371.08
	SQ	323.0	519.0	410.09	304.0	457.0	370.65	307.0	455.0	372.18
60%	FP	304.0	467.0	374.83	304.0	434.0	364.66	304.0	458.0	369.3
	HP	319.0	471.0	395.32	304.0	434.0	364.66	321.0	470.0	382.92
	DD	327.0	481.0	404.03	304.0	434.0	364.66	323.0	470.0	387.09
	SQ	327.0	514.0	415.71	304.0	434.0	364.66	315.0	502.0	394.73
80%	FP	313.0	475.0	378.35	304.0	462.0	376.67	313.0	466.0	372.92
	HP	323.0	491.0	398.98	304.0	462.0	376.67	320.0	484.0	384.82
	DD	325.0	487.0	407.63	304.0	462.0	376.67	323.0	462.0	388.17
	SQ	327.0	514.0	420.96	304.0	462.0	376.67	322.0	467.0	394.41
100%	FP	304.0	467.0	378.59	304.0	467.0	378.4	304.0	467.0	377.82
	HP	323.0	491.0	400.03	304.0	467.0	378.4	323.0	482.0	386.74
	DD	328.0	487.0	407.86	304.0	467.0	378.4	322.0	457.0	390.39
	SQ	327.0	528.0	421.98	304.0	467.0	378.4	321.0	477.0	396.4

Table A.4: Summary Statistics of **activities modes investment** for each UBL, topology and solution strategy.

UBL	Top.	Min-Max			Indep. Dec.			Min-Max Lim.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	FP	20.0	332.0	136.21	0.0	207.0	76.08	0.0	205.0	80.76
	HP	20.0	405.0	163.61	0.0	207.0	76.08	0.0	253.0	75.62
	DD	20.0	372.0	173.96	0.0	207.0	76.08	0.0	212.0	73.02
	SQ	42.0	391.0	183.35	0.0	207.0	76.08	0.0	230.0	73.12
60%	FP	52.0	332.0	174.43	64.0	405.0	181.84	52.0	332.0	155.7
	HP	67.0	338.0	197.57	64.0	405.0	181.84	64.0	338.0	153.28
	DD	73.0	405.0	211.09	64.0	405.0	181.84	31.0	360.0	150.6
	SQ	64.0	475.0	220.39	64.0	405.0	181.84	31.0	399.0	145.36
80%	FP	36.0	340.0	178.69	32.0	332.0	164.76	36.0	340.0	158.44
	HP	85.0	380.0	203.51	32.0	332.0	164.76	36.0	340.0	151.96
	DD	100.0	413.0	218.65	32.0	332.0	164.76	20.0	327.0	149.11
	SQ	86.0	430.0	225.11	32.0	332.0	164.76	20.0	313.0	144.66
100%	FP	36.0	340.0	176.55	36.0	340.0	176.5	36.0	340.0	175.62
	HP	77.0	380.0	199.46	36.0	340.0	176.5	36.0	340.0	163.73
	DD	77.0	380.0	215.33	36.0	340.0	176.5	31.0	327.0	160.64
	SQ	77.0	430.0	223.2	36.0	340.0	176.5	31.0	313.0	155.79

Table A.5: Summary Statistics of **mitigations investment** for each UBL, topology and solution strategy.

UBL	Top.	Min-Max			Indep. Dec.			Min-Max Lim.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	FP	316.0	733.0	505.56	342.0	603.0	446.73	316.0	581.0	438.67
	HP	332.0	842.0	551.82	342.0	603.0	446.73	332.0	601.0	443.35
	DD	361.0	832.0	572.83	342.0	603.0	446.73	338.0	600.0	444.1
	SQ	401.0	882.0	593.44	342.0	603.0	446.73	338.0	603.0	445.3
60%	FP	389.0	746.0	549.26	415.0	822.0	546.5	389.0	746.0	525.0
	HP	426.0	803.0	592.89	415.0	822.0	546.5	412.0	803.0	536.2
	DD	431.0	852.0	615.12	415.0	822.0	546.5	401.0	810.0	537.69
	SQ	423.0	925.0	636.1	415.0	822.0	546.5	412.0	819.0	540.09
80%	FP	398.0	796.0	557.04	405.0	745.0	541.43	398.0	744.0	531.36
	HP	429.0	811.0	602.49	405.0	745.0	541.43	405.0	743.0	536.78
	DD	442.0	854.0	626.28	405.0	745.0	541.43	405.0	744.0	537.28
	SQ	460.0	903.0	646.07	405.0	745.0	541.43	405.0	744.0	539.07
100%	FP	394.0	796.0	555.14	394.0	754.0	554.9	394.0	754.0	553.44
	HP	429.0	806.0	599.49	394.0	754.0	554.9	394.0	753.0	550.47
	DD	448.0	811.0	623.19	394.0	754.0	554.9	393.0	749.0	551.03
	SQ	471.0	909.0	645.18	394.0	754.0	554.9	390.0	754.0	552.19

Table A.6: Summary Statistics of **total investment** for each UBL, topology and solution strategy.

UBL	Top.	Min-Max			Indep. Dec.			Min-Max Lim.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	FP	87.0	314.0	179.56	122.0	326.0	218.49	106.0	326.0	212.08
	HP	81.0	314.0	164.17	105.0	326.0	217.28	129.0	314.0	213.81
	DD	76.0	314.0	160.41	105.0	326.0	214.13	132.0	314.0	213.87
	SQ	76.0	247.0	155.93	105.0	326.0	209.53	115.0	314.0	215.55
60%	FP	87.0	345.0	198.57	95.0	383.0	208.64	96.0	345.0	211.2
	HP	87.0	329.0	185.74	95.0	383.0	207.66	105.0	343.0	213.36
	DD	95.0	320.0	178.38	95.0	383.0	206.83	100.0	361.0	215.95
	SQ	87.0	320.0	173.71	95.0	364.0	206.41	107.0	361.0	218.74
80%	FP	96.0	369.0	209.68	96.0	353.0	226.59	96.0	369.0	223.13
	HP	96.0	369.0	196.51	96.0	353.0	226.52	96.0	369.0	228.53
	DD	96.0	346.0	189.6	96.0	353.0	226.39	96.0	351.0	232.32
	SQ	94.0	346.0	186.32	96.0	353.0	226.01	103.0	357.0	235.71
100%	FP	102.0	373.0	212.98	102.0	373.0	212.85	102.0	373.0	213.44
	HP	102.0	354.0	200.68	102.0	373.0	212.85	102.0	373.0	221.99
	DD	102.0	350.0	194.1	102.0	373.0	212.85	102.0	373.0	226.35
	SQ	94.0	350.0	189.86	102.0	373.0	212.85	109.0	387.0	231.01

Table A.7: Summary Statistics of **activities costs due to risk impacts** for each UBL, topology and solution strategy.

UBL	Top.	Min-Max			Indep. Dec.			Min-Max Lim.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	FP	0.0	194.0	72.99	40.0	328.0	156.13	36.0	257.0	126.54
	HP	0.0	243.0	63.2	51.0	624.0	248.52	23.0	495.0	178.06
	DD	0.0	327.0	47.08	96.0	904.0	386.85	16.0	698.0	229.47
	SQ	0.0	225.0	34.06	96.0	984.0	458.88	7.0	738.0	284.96
60%	FP	0.0	186.0	69.93	12.0	236.0	104.91	8.0	186.0	85.13
	HP	0.0	325.0	61.11	36.0	457.0	179.42	6.0	355.0	113.88
	DD	0.0	408.0	51.13	54.0	676.0	272.8	0.0	498.0	136.59
	SQ	0.0	287.0	43.62	54.0	894.0	329.29	0.0	598.0	170.09
80%	FP	0.0	183.0	70.53	4.0	250.0	90.32	4.0	239.0	87.11
	HP	0.0	321.0	61.18	8.0	504.0	157.2	0.0	402.0	122.15
	DD	0.0	401.0	51.31	8.0	734.0	237.93	0.0	566.0	148.6
	SQ	0.0	382.0	45.18	8.0	864.0	288.91	0.0	623.0	185.94
100%	FP	0.0	183.0	71.53	0.0	183.0	71.9	0.0	183.0	72.77
	HP	0.0	321.0	63.17	0.0	465.0	128.13	0.0	386.0	108.6
	DD	0.0	399.0	53.36	0.0	645.0	202.05	0.0	554.0	131.78
	SQ	0.0	362.0	46.54	0.0	707.0	246.29	0.0	570.0	165.67

Table A.8: Summary Statistics of **penalties costs** for each UBL, topology and solution strategy.

UBL	Top.	Min-Max			Indep. Dec.			Min-Max Lim.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
40%	FP	580.0	1039.0	758.11	591.0	1157.0	821.35	591.0	1095.0	777.29
	HP	596.0	1114.0	779.19	652.0	1424.0	912.53	624.0	1224.0	835.22
	DD	592.0	1192.0	780.32	660.0	1673.0	1047.71	605.0	1432.0	887.44
	SQ	594.0	1145.0	783.43	660.0	1753.0	1115.14	625.0	1458.0	945.81
60%	FP	621.0	1146.0	817.76	641.0	1162.0	860.05	621.0	1146.0	821.33
	HP	632.0	1244.0	839.74	681.0	1328.0	933.58	638.0	1244.0	863.44
	DD	631.0	1344.0	844.63	689.0	1498.0	1026.13	638.0	1367.0	890.23
	SQ	632.0	1311.0	853.43	691.0	1625.0	1082.2	638.0	1379.0	928.92
80%	FP	632.0	1170.0	837.25	641.0	1198.0	858.34	632.0	1172.0	841.6
	HP	650.0	1272.0	860.18	657.0	1358.0	925.15	650.0	1284.0	887.46
	DD	649.0	1371.0	867.19	657.0	1588.0	1005.75	650.0	1428.0	918.2
	SQ	650.0	1332.0	877.57	661.0	1718.0	1056.35	652.0	1464.0	960.72
100%	FP	632.0	1172.0	839.65	632.0	1172.0	839.65	632.0	1172.0	839.65
	HP	656.0	1274.0	863.34	656.0	1300.0	895.88	656.0	1281.0	881.06
	DD	655.0	1371.0	870.65	656.0	1480.0	969.8	656.0	1412.0	909.16
	SQ	656.0	1376.0	881.58	658.0	1490.0	1014.04	658.0	1432.0	948.87

Table A.9: Summary Statistics of **total cost** for each UBL, topology and solution strategy.

A.2
Solution Strategy Performance Assessment

A.2.1
Instance Parameters

In Table A.10, we present minimum, maximum and average values of model parameters from all generated instances of Section 5.3.2. As previously mentioned, we omitted c_a^1, c_a^2 and h_r^1, as $c_a^1 = d_a^1$, $c_a^2 = 2 \cdot c_a^1$ and $h_r^1 = 0$.

Param.	**Min.**	**Max.**	**Avg.**
τ_p	32	199	96.82
ρ_p	1	19	7.73
d_a^1	1	10	7.96
d_a^2	1	8.0	5.54
σ_r^1	30.0	95.0	64.76
σ_r^2	51.0	99.0	85.11
Δ_{ar}^1	1	10	5.05
Δ_{ar}^2	0	7	1.77
θ_{ar}^1	1	36	10.36
θ_{ar}^2	0	23	4.02
h_r^2	2	147	26.02
β	564.6	1069.8	817.06

Table A.10: Summary Statistics of instance parameters.

A.2.2 Solution Statistics

In Tables A.11 and A.12, we present solution statistics for the experiments of Section 5.3.2. They show minimum, maximum and average values of the selected metrics for instances having portfolios with the same number of activities and risks. The tables' columns are defined as:

- **NR**: number of risks of the portfolio;
- **NA**: number of activities of the portfolio;
- **Act. Mod. Inv.**: the investment on activities modes ($\sum_{a\in A}\sum_{m\in M_a} c_a^m x_a^m$);
- **Mit. Inv.**: the investment in mitigations ($\sum_{r\in R}\sum_{l\in L_r} h_r^l y_l^r$);
- **Tot. Inv.**: the total investment ($\sum_{a\in A}\sum_{m\in M_a} c_a^m x_a^m + \sum_{r\in R}\sum_{l\in L_r} h_r^l y_l^r$);
- **Risk Imp. Cost**: the total cost impact in activities due to materialized risks ($\sum_{a\in A}\sum_{r\in R_a}\sum_{l\in L_r} \theta_{ar}^l z_r^l$);
- **Pen. Cost**: the total cost impact due to penalties for delays ($\sum_{p\in P} \rho_p o_p$);
- **Tot. Cost**: the portfolio total cost ($\sum_{a\in A}\sum_{m\in M_a} c_a^m x_a^m + \sum_{r\in R}\sum_{l\in L_r} h_r^l y_l^r + \Pi(x, y)$).

NR	NA	Act. Mod. Inv.			Mit. Inv.			Tot. Inv.		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
12	48	409.0	554.0	483.3	86.0	288.0	174.2	495.0	819.0	657.5
	64	533.0	655.0	599.3	80.0	265.0	198.0	613.0	879.0	797.3
	80	655.0	800.0	719.4	81.0	258.0	160.8	790.0	980.0	880.2
16	48	425.0	542.0	481.9	194.0	362.0	266.0	653.0	855.0	747.9
	64	553.0	721.0	654.6	176.0	401.0	298.8	794.0	1097.0	953.4
	80	660.0	836.0	736.2	108.0	397.0	230.3	792.0	1233.0	966.5
20	48	410.0	564.0	492.2	303.0	523.0	412.6	728.0	1014.0	904.8
	64	543.0	785.0	669.4	112.0	653.0	367.9	759.0	1438.0	1037.3
	80	716.0	873.0	784.4	147.0	470.0	314.3	890.0	1265.0	1098.7

Table A.11: Summary Statistics of **activities modes investment**, **mitigations investment** and **total investment**.

NR	NA	Risk Imp. Cost			Pen. Cost			Tot. Cost		
		Min.	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**	**Min.**	**Max.**	**Avg.**
12	48	125.0	275.0	180.9	0.0	73.0	20.8	712.0	1037.0	859.2
	64	105.0	264.0	179.1	0.0	66.0	15.2	790.0	1103.0	991.6
	80	103.0	312.0	185.9	0.0	188.0	21.3	935.0	1364.0	1087.4
16	48	125.0	253.0	197.0	0.0	270.0	74.6	845.0	1266.0	1019.5
	64	156.0	324.0	236.0	0.0	81.0	32.7	1026.0	1408.0	1222.1
	80	166.0	332.0	246.1	0.0	75.0	11.9	1046.0	1508.0	1224.5
20	48	180.0	421.0	299.9	0.0	414.0	180.5	922.0	1753.0	1385.2
	64	230.0	366.0	290.9	0.0	513.0	148.7	1158.0	1890.0	1476.9
	80	223.0	416.0	334.5	0.0	104.0	48.9	1215.0	1663.0	1482.1

Table A.12: Summary Statistics of **risk impacts in cost**, **penalties cost** and **total cost**.

B
Model Extensions

B.1
Minimizing Worst-Case Sum of Project Durations

This variant of the models presented in Chapter 5, was motivated by the planning phase of the case study described in Chapter 6. The idea was to develop a robust plan for the projects that compose the portfolio, while at the same time aiming completions as earlier as possible. The solution of the model was used to establish the initial execution plan and project deadlines, providing the necessary protection against the uncertainty.

Given the models and definitions of Section 5.2, the necessary adaptation needed, is to remove the cost factors from the objective functions of first and second levels. Next, we present the resulting models after performing this adaptation:

$$\underset{x,y}{\text{Min}} \quad \Pi(x,y) \tag{B-1}$$

$s.t.$

$$\sum_{a \in A} \sum_{m \in M_a} c_a^m x_a^m + \sum_{r \in R} \sum_{l \in L_r} h_r^l y_l^r \leq \alpha \tag{B-2}$$

$$\sum_{m \in M_a} x_a^m = 1 \quad \forall a \in A \tag{B-3}$$

$$\sum_{l \in L_r} y_r^l = 1 \quad \forall r \in R \tag{B-4}$$

$$x_a^m \in \{0,1\} \quad \forall a \in A, m \in M_a \tag{B-5}$$

$$y_r^l \in \{0,1\} \quad \forall r \in R, l \in L_r \tag{B-6}$$

$$\Pi(x,y) = \max_{z,u,zu} \sum_{p\in P}\sum_{(i,j)\in E}\Big[\sum_{m\in M_i} d_i^m x_i^m u_{ij}^p + \sum_{r\in R_i}\sum_{l\in L_r}\Delta_{ir}^l zu_{rl}^{ijp}\Big] \tag{B-7}$$

$s.t.$

$$z_r^l \leq y_r^l \quad \forall r\in R, l\in L_r \tag{B-8}$$

$$\sum_{r\in R}\sum_{l\in L_r}\sigma_r^l z_r^l \leq \beta \tag{B-9}$$

$$\sum_{j\in\delta_i^+} u_{ij}^p = 1 \quad \forall p\in P,\ i=0 \tag{B-10}$$

$$\sum_{i\in\delta_j^-} u_{ij}^p = 1 \quad \forall p\in P,\ j=s_p \tag{B-11}$$

$$\sum_{i\in\delta_a^-} u_{ia}^p - \sum_{j\in\delta_a^+} u_{aj}^p = 0 \quad \forall p\in P,\ a\in A,\ a\notin\{0,s_p\} \tag{B-12}$$

$$zu_{rl}^{ijp} \leq z_r^l \quad \forall p\in P,(i,j)\in E, r\in R_i, l\in L_r \tag{B-13}$$

$$zu_{rl}^{ijp} \leq u_{ij}^p \quad \forall p\in P,(i,j)\in E, r\in R_i, l\in L_r \tag{B-14}$$

$$zu_{rl}^{ijp} \geq z_r^l + u_{ij}^p - 1 \quad \forall p\in P,(i,j)\in E, r\in R_i, l\in L_r \tag{B-15}$$

$$z_r^l \in\{0,1\} \quad \forall r\in R, l\in L_r \tag{B-16}$$

$$u_{ij}^p \in\{0,1\} \quad \forall (i,j)\in E, p\in P \tag{B-17}$$

For the risk-scenario enumeration model, the adaptation is to constrain the values of variable T to the maximum sum of project durations between all scenarios of RS. After carrying out this adaptation, the resulting model follows:

$$\min_{x,y,o,f} \quad T \tag{B-18}$$

$s.t.$

$$T \geq \sum_{p\in P} f_p^\xi \quad \forall \xi\in RS \tag{B-19}$$

$$f_p^\xi = LP(x,y,\xi,p) \quad \forall \xi\in RS,\ p\in P \tag{B-20}$$

$$(B-2), (B-3), (B-4), (B-5) \text{ and } (B-6)$$

From this point, the function $LP(x,y,\xi,p)$ can be replaced by its dual and incorporated in the previous model to obtain the one-level mixed integer programming final model. The dual of LP is detailed in Section 5.2. The adaptation of the cut-generation algorithm of Section 5.2.4 is straightforward.

B.2 Multi-Budget Uncertainty Set

The motivation of this extension is to model an uncertainty set for a qualitative risk analysis mapping. In this context, each risk is assigned to a probability category (e.g., low, medium and high), instead of given a numeric probability estimate. Despite loosing precision in the quantification of risk probabilities, it is easier to place them into categories and also communicate results from possible analysis.

We propose to define an uncertainty set that limits the number of risks that could be materialized by the adversary at each probability category. Given the set of probability categories Ψ, for each impact mode l of a risk r, a binary parameter σ_{rl}^{ψ} that indicates if the risk r in impact mode l belongs to the category $\psi \in \Psi$ is defined. For each probability category ψ, an uncertainty budget β_{ψ} is also defined. To model the referred uncertainty set, constraint 5-9 should be replaced by the following set of constraints:

$$\sum_{r \in R} \sum_{l \in L_r} \sigma_{rl}^{\psi} z_r^l \leq \beta_{\psi} \quad \forall \psi \in \Psi \tag{B-21}$$

Despite to model a qualitative mapping of risks be the motivation to the proposal of this extension, it also represents a general multi-budget uncertainty set, where values of σ_{rl}^{ψ} and β_{ψ} may not be limited to belong to $\{0, 1\}$ or $\mathbb{Z}$ respectively.