2
Basic concepts and literature review

Only recently, researchers started applying enrichment techniques to the modelling of interaction between hydraulic and natural fractures. Consequently, the amount of research in this specific subject is limited. Nevertheless, a wide variety of research related to different parts of this subject is available, such as the use of enrichment techniques to simulate multiple fractures in uncoupled problems, or the modelling of single planar hydraulic fractures. To cover all the areas of knowledge that are involved in this thesis, three main subjects were deeply reviewed. The first section presents a review on hydraulic fracture modelling, from its early analytical works based on linear elastic fracture mechanics to modern numerical techniques. Next, studies regarding the interaction between fractures, physical or numerical, are contextualized and presented. The last section approaches the history of the eXtended Finite Element Method and its applications to coupled or branched problems.

2.1.
Hydraulic fracture modelling

2.1.1.
Introduction

An idealized plot of a borehole pressure response against injected volume is represented in Figure 2.1a. The first linear part represents the system elastic deformation, mainly the fracturing fluid compression in the borehole. Fracture initiation is identified by a pressure peak, followed by a drastic pressure drop (breakdown), which means the fracture volume grows at a higher rate than the injected volume. Keeping continuous pumping will lead to stable fracture propagation. In a second pumping cycle (Figure 2.1a), a reduction of the peak pressure is noted. Once the fracture already exists, no tensile strength has to be reached and the in-situ stresses are different from the ones before the first cycle.

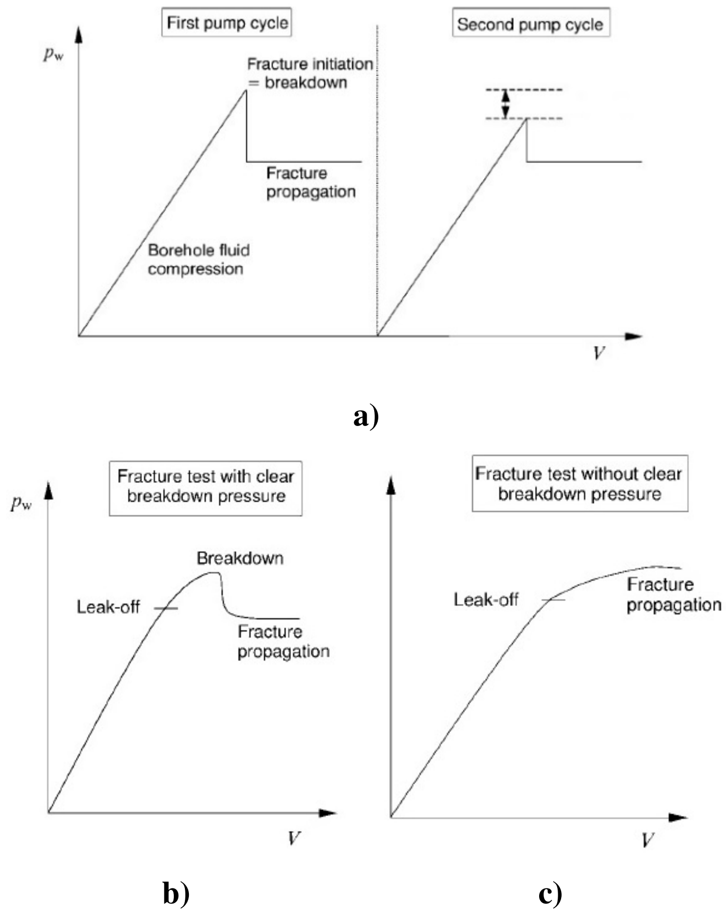


Figure 2.1 – Borehole pressure response during hydraulic fracture of a vertical wellbore (Fjaer, 2008). a) Idealized plot of two pressure cycles. b) Realistic plot with distinct breakdown pressure. c) Realistic plot without distinct breakdown pressure

More realistic plots show this processes occur with smoother transitions which are harder to detect. Figure 2.1b shows a test with clear breakdown pressure, while in Figure 2.1c it is not detectable. According to Fjaer et al. (2008) this can happen due to several reasons, from filter cake efficiency to plastification and stress dependent properties, temperature and leakage.

Hydraulic fracture propagation is mainly based on three physical processes. First, fluid flow within the fracture, which imposes a pressure load on the fracture surfaces. Second, rock mechanical deformation as a result of the flow pressure. Third, fracture propagation when a critical condition is reached. Furthermore, other complex phenomena may be involved, such as:

- leak-off of fracturing fluid from the fracture to the rock matrix;
- transport of suspended proppant particles within the fracture;

- effects of temperature on the fracturing fluid rheology;
- effects of chemical composition of the fluid on rock behaviour.

Hydraulic fracture treatments usually take place in a time-scale of tens of minutes to a few hours, depending upon the designed fracture size and volume of proppant to be placed (Adachi *et al.*, 2007). As the body wave speeds are much larger than the macroscopic propagation velocity of a hydraulically driven fracture, it may be admitted that dynamic effects like wave propagation can be neglected and fracture propagation can be analysed as a succession of equilibrium states. For example, a fracture is driven in the order of 610 m in 5 to 8 hours, while an elastic wave traverses this distance in 200 milliseconds (Hanson, Shaffer and Anderson, 1981).

The first attempts of modelling the hydraulic fracturing process date from more than 50 years. However, it remains a current challenge, not only due to the wide variety of phenomena and scales associated but also because it is so hard to track evidences in the laboratory or in the field. Standing before a large number of models created to model different situations and phenomena, one has to choose which model to use based mainly on experience with the reservoir characteristics.

According to Valkó and Economides (1995), a hydraulic fracturing model should follow three basic principles:

- Fundamental laws such as material and energy balances must be obeyed;
- Complete mathematical formulation of the governing and boundary equations, without resorting to “weighing factors”, should be derived;
- A fracture tip propagation criterion and its interaction with the provided energy must be explicitly stated.

In general, the solution for hydraulic fracture modelling consists of a series of "snapshots" that correspond to unique instances in time and crack shape. From the literature review made throughout this research, two different philosophies of modelling hydraulic fracturing could be differentiated.

The first has its domain in the plane where the fracture grows, with the width of the fracture being perpendicular to the calculation plane. This is a classical approach that assumes a bi-planar fracture, widely used by the industry to design or simulate the process.

The second approach models the continuum space. It can be used in 2D or 3D models, and the fracture is usually simulated in a perpendicular plane to the fracture plane in case of two-dimensional simulations. In these models, the propagation direction does not have to be known at start and can change during its development. These have been used mostly to evaluate the interaction between hydraulic fractures and natural fractures. Although only a few references with employment of these models in the industry were found, those models have high academic interest for the capabilities they present.

The transport and placement of proppant within the fracture is usually modelled by representing the slurry (i.e., the mixture of proppant and fluid) as a two-component, interpenetrating continuum. This implies that the fluid flow equations are solved for the mixture, and not for each individual component. Modelling of the proppant transport then reduces to solving an advective (mass conservation) equation for the proppant volumetric concentration. More complex modelling of this phenomena can be found in the literature, including models used in commercial simulators (Kresse *et al.*, 2014) that assume three layer models (proppant, slurry and clean fluid).

In a different process from the conventional hydraulic fracturing, named Thermally Induced Fracturing (TIF), thermal effects take a major role in the treatment results, especially when there is a large temperature difference between the (cold) injection water and the (hot) reservoir. Typical response is a sudden increase in injectivity after a significant period of stable injection. This reflects that the reservoir rock has been gradually cooled during injection of the cold water. The reservoir rock shrinks due to cooling, and eventually the smallest in situ stress is reduced to a level below the bottom hole injection pressure. This results in the creation of a fracture which provides a much larger contact area with the formation and hence dramatic increase in injectivity (Fjaer, 2008). Cohen, Kresse and Weng (2013) studied the impact of the reservoir temperature on the production for different fracturing fluids with their rheological properties depending on temperature, through the implementation of a model that couples the heat transport equations inside the fracture and the heat exchange with the reservoir.

Chemical effects may also exist in fracturing treatments such as acid fracturing, which is used in very specific cases (shallow carbonates), and consists on using acid instead of proppant. This will react with the rock, creating channels that improve the permeability of the fracture. In this field, studies exist to predict the influence of chemical solutions on rock properties, such as in Karfakis and Akram (1993).

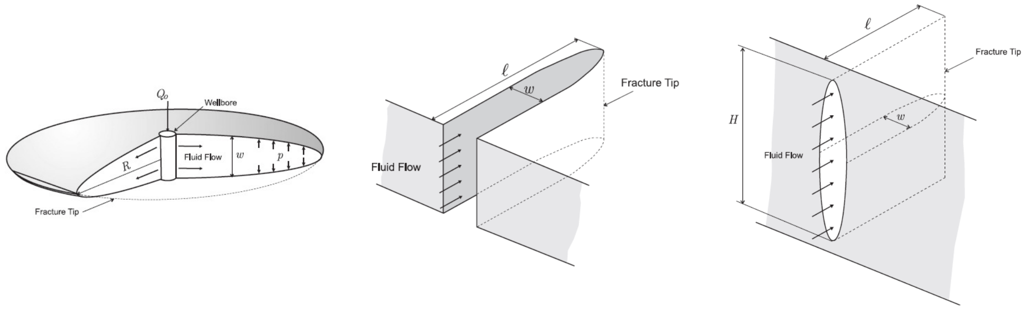
The research papers presented in this literature review are focused on the phenomena which are considered of main importance to hydraulic fracture modelling, i.e. fluid flow, mechanical behaviour and fracture propagation.

2.1.2.
Analytical models

A basic stimulation treatment simulation needs to return parameters which are essential to study the effectiveness of the treatment, as pumping time, pressure in the well and fracture volume (width, height and length). Therefore, the first efforts to model the process tried to couple basic phenomena such as:

- elasticity of the rock medium around the fracture;
- fracturing fluid flow, to relate the injection with pressure inside the fracture;
- material balance, to relate the fracture growth with injected volume;
- proppant propagation.

Three analytical solutions proved to be reliable enough to be used for decades as basis for hydraulic fracturing prediction. First, Sneddon (1946) developed solutions for geometries that conform with a plane strain fracture or a “penny-shaped” fracture with radial flow (see Figure 2.2a). In the “penny-shaped” model, the fracture width is determined by assuming a uniform fracture flow pressure. This solution applies best when the well orientation coincides with the direction of minimum confining stresses, i.e. the fracture evolves around the wellbore.



a) **b)** **c)**

Figure 2.2 – Schematic of fracture geometry of analytical solutions: a) Penny shaped. b) KGD. c) PKN (Adachi et al., 2007)

Khristianvic and Zheltov (1955) and Geertsma and de Klerk (1969) developed the KGD analytical model for hydraulic fracturing, in which a plane strain geometry is admitted in the vertical direction (see Figure 2.2b). This way, the fracture width is constant and the flow channel (from the wellbore to the fracture) is rectangular. Furthermore, the flow rate is constant and all cross sections are independent along the vertical direction. This model shows better applicability to short fractures with large height. However, as pointed by Valkó and Economides (1995) it tends to overestimate the fracture width. Another limitation is that net pressure decreases with time and is independent of injection rate, which contradicts experience.

Perkins and Kern (1961) introduced an analytical model which was improved by Nordgren (1972) to account for the effect of fluid leak-off into the surrounding rock mass, resulting in the PKN model (see Figure 2.2c). In this model, the fracture height is fixed and admitted to be much smaller than the fracture length. For each vertical plane, an elliptical shape is defined and it is assumed that plane strain occurs. However, the stresses and deformations are different in each vertical plane. The flow channel is elliptical and the flow rate is constant, assuming uniform pressure proportional to the fracture width in vertical direction. This method provides good approximations for elongated fractures with shorter heights.

Both PKN and KGD models are based on the assumptions that fracture height is constant while the other dimensions in width and length increase during propagation. The key difference between these methods is the way of considering fracture width variation along vertical and horizontal directions. This difference leads to two different ways of solving the hydraulic fracturing problem. In the PKN model, the effect of the fracture tip is not considered so that the stress concentration is controlled by the effect of viscous fluid flow. In the KGD model, however, the stress concentration at the fracture tip is more important for the fracture propagation (Youn, 2016).

These analytical solutions are limited to analyse very simple geometries in a homogeneous and isotropic medium, but they provide fundamental understanding about the asymptotic behaviour of the fracture tip (Youn, 2016). The strong dependence of the solution on the asymptotic behaviour of the tip led several authors to propose analytical solutions to cover different propagation regimes. Desroches et al. (1993) studied the propagation on zero-toughness and impermeable formations. Lenoach (1995) suggested the zero-toughness and leak-off dominated case, and Detournay and Garagash (2003) and Bunger, Detournay and Garagash (2005) dealt with toughness-dominated regimes. These studies have showed that hydraulic fractures may be controlled by toughness, viscosity, or fluid leak-off and that the fracture regime may change while the fracture evolves (Detournay, 2004; Adachi and Detournay, 2008).

2.1.3. Numerical models

2.1.3.1. Modelling bi-planar fractures

For a wide variety of treatments and reservoir characteristics, it is common to assume the facture as planar, perpendicular to the minimum confining stress. As stated by Adachi, et al. (2007) simple computer models were developed using the KGD and PKN geometries with proppant transport. These served as guides in the treatment design and provided a method to show the sensitivity to critical input parameters of injection rate, treatment volumes, fluid viscosity and leak-off, and provided a basis for changing these parameters to increase the propped fracture penetration and also to minimize proppant bridging and screen-outs. One should notice that these models (PKN and KGD) were a base for several variations, such as the introduction of the Carter Equation for leak-off (PKN-C and KGD-C) and the consideration of a power law for non-newtonian fluids (PKN-α and KGD-α), as presented by Valkó and Economides (1995).

Since the 80s, these simpler models have been developed to become more flexible and able to adapt to more realistic problems, such as multi-layer reservoirs, variable injection rates and variation of the three dimensions of the fracture (width, length, height), being usually called Pseudo 3D (P3D) models (Settari, 1988; Meyer, 1989; Warpinski and Smith, 1989). These usually assume the sub-division of the fracture length in cells with different heights, allowing the growth of height and length, computed separately and based on KGD and PKN solutions, respectively. The reservoir elastic properties are considered homogeneous and Linear Elastic Fracture Mechanics governs fracture propagation. Leak-off is generally assumed as unidimensional and in the fracture plane flow is usually computed by the Finite Differences Method in one (length) or two directions.

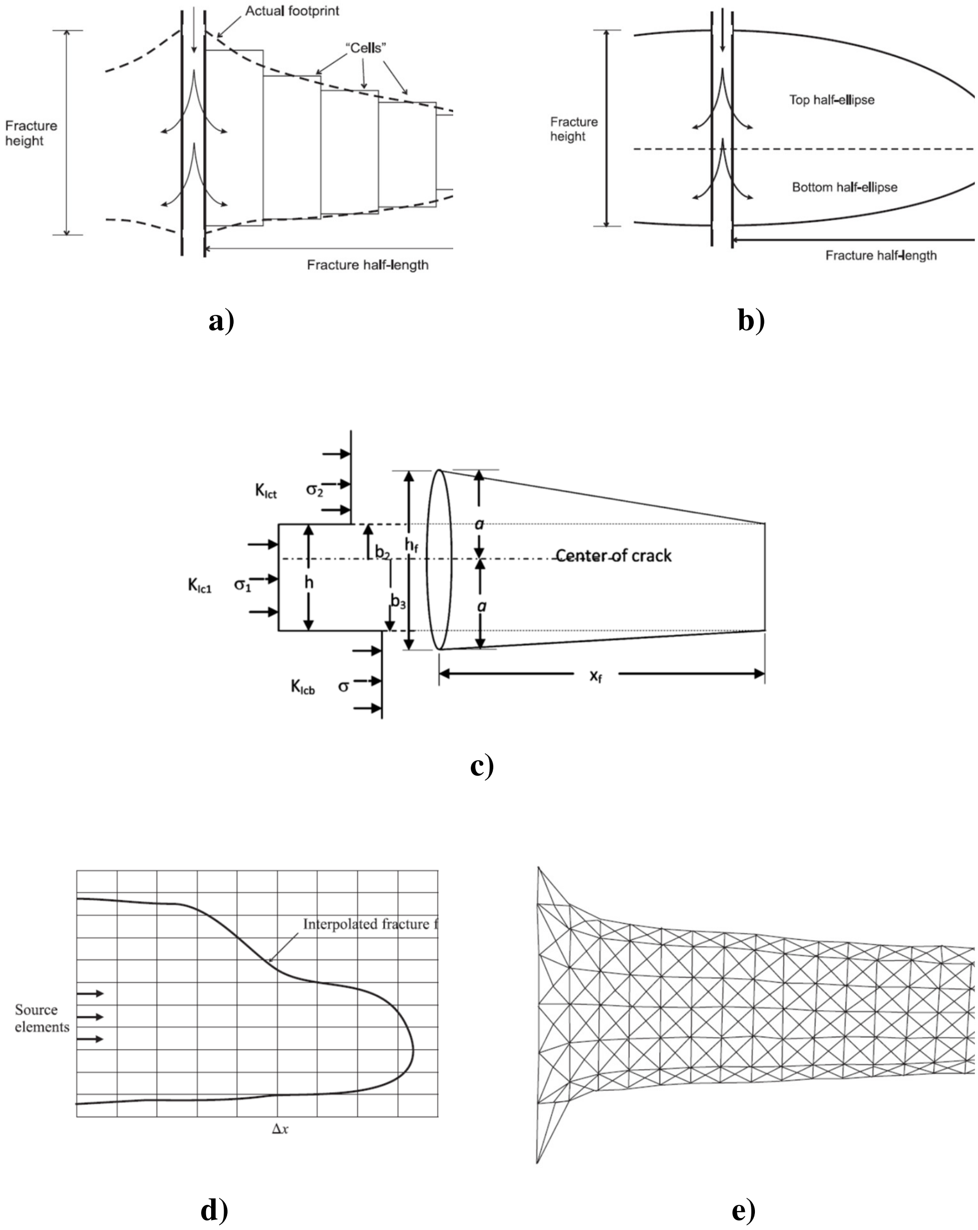


Figure 2.3 – Examples of Pseudo 3D (P3D) and Planar 3D (PL3D) models: a) P3D cell based (Settari, 1988). b) P3D cell based (Meyer, 1989). c) P3D cell based (Warpinski and Smith, 1989). d) PL3D with fixed quadrangular mesh (Clifton and Abou-Sayed, 1981). e) PL3D with moving triangular mesh (Clifton and Abou-Sayed, 1981)

Although more complex than the analytical models and still CPU inexpensive, these methods are limited to certain variations of geometry and do not consider geometric variations in a 3D space.

More complete models were also proposed, such as the Planar 3D (PL3D) model (Clifton and Abou-Sayed, 1981). In this, it is assumed that the fracture footprint and the coupled fluid flow equation are described by a 2D mesh of cells oriented in a vertical plane and full 3D elasticity equations are used to describe width as function of fluid pressure. Figure 2.3 shows some examples of the mentioned models.

2.1.3.2. Modelling in the continuum

There is a rising tide of evidence from direct monitoring of actual field treatments that suggests that fractures grow in a complicated manner, taking advantage of local heterogeneities and layering. These factors complicate the design of treatments and make numerical modelling far more challenging (Adachi *et al.*, 2007). It should be noted as well that many of the current hydraulic fracturing simulators do not predict the correct wellbore fluid pressure even for planar fractures (Carter *et al.*, 2000).

Numerous 2D, pseudo-3D, and planar 3D hydraulic fracturing simulators work relatively well in many cases where the geometry of the fracture is easily defined and constrained to a single plane. However, there are instances where a fully 3D simulator is necessary for more accurate modelling. Furthermore, many hydraulic fracturing operations are performed in soft formations that are prone to non-linear mechanical failure—a real challenge for current models that are based on the principles of Linear Elastic Fracture Mechanics (LEFM). A fully 3D simulation is also essential to understand the behaviour of a hydraulic fracture in a reservoir with natural fractures, as well as the reciprocal influence of fractures when the treatment comprises multiple injections.

In reviewed literature, many different methods to simulate 2D or 3D fracture propagation in the continuum were found, mostly being based on a system of differential equations to be solved applying numerical methods. The transient effect in time is almost always admitted as quasi-static, with the solution obtained in each time step being dependent of the previous time step using Finite Difference or Newmark schemes.

It may be affirmed that the Finite Element Method is the most popular, but other approaches were also successfully applied, such as the Boundary Element Method, the Discrete Element Method or the Finite Differences Method. Although more often complex assumptions about crack tip behaviour are being made, such as damage and cohesive laws, the LEFM approach is still used in many studies.

Finite Element Method

The Finite Element Method (FEM) is usually applied to either the mechanical deformations, the fluid flow or the coupling of both phenomena. When coupled, it is very common to use Biot's Theory and when applied to fluid flow in separate, the lubrication equation is solved.

For mechanical behaviour and discretization of the fracture, the following groups of element formulations may be highlighted:

- Interface elements with cohesive laws (cohesive elements);
- Models based on concept of damage mechanics;
- Plastic flow models;
- Elements with enriched nodes (eXtended finite element method).

In the first group, the fracture path is an input for the model, as the interface elements have to be placed according to the discontinuity position. In the remaining groups, the crack path is a solution of the problem and complex geometries may be obtained, although it depends on several factors, such as mesh geometry and refinement.

Interface elements with cohesive laws are easy to implement and avoid stress and pressure singularities at crack tip. In addition, the cohesive zone model has the interesting capability of modelling microstructural damage mechanisms inherent to hydraulic fracturing such as initiation of micro cracking and coalescence (Chen *et al.*, 2009). However, Shojaei, Dahi Taleghani and Li (2014) mentioned some limitations of the cohesive models, such as difficulties in situations involving intersecting discontinuities or the inability to predict changes in rock poroelastic properties like Biot coefficient and Biot modulus.

Carrier and Granet (2012) used interface elements with a cohesive law and the hydro-mechanical equations in a fully coupled approach to simulate four limiting propagation regimes: toughness-fracture storage, toughness-leak-off, viscosity-fracture storage and viscosity-leak-off dominated. Similarly, Bendezu *et al.* (2013) used cohesive elements to successfully compare fracture propagation with analytical solutions for toughness-dominated hydraulic fractures in KGD and

Penny-shaped geometries. Chen *et al.* (2009) also compared models with cohesive elements and reported excellent agreement between the finite element results and analytical solutions for the case where the fracture process is dominated by rock fracture toughness. Papanastasiou (1999) used cohesive elements, together with a remeshing scheme was employed with fine mesh near the fracture tip during propagation, to evaluate of the effective fracture toughness of the material, assuming elasto-plastic behaviour for the rock medium.

Yao *et al.* (2010) ran three-dimensional models of a three-layer water injection case to compare with a P3D model, PKN model and analytical solution. Their results showed that, compared with the traditional methods, the models with cohesive elements can predict the hydraulic fracture geometry more accurately. Complex 3D models were also applied to real case studies, such as a staged fracturing process of a horizontal well in the Daqing Oilfield, China (Zhang *et al.*, 2010).

Models based on the concept of damage mechanics were also used to simulate hydraulic fracturing. Shojaei, Dahi Taleghani and Li (2014) pointed some advantages of continuum damage based porous models, such as their capability to capture crack initiation, propagation, interaction and possible branching in an integrated framework, allowing material properties evolution during failure. Another main advantage lies in the fact that common continuum elements are used and there is no need to remesh, once the fractured elements have their properties weakened when fracturing occurs, i.e. once a certain value of damage occurs, the corresponding element is removed from the model by setting a small value for its elastic modulus. Li *et al.* (2012) used the same concept to simulate hydraulic fracturing on a laboratory sample, with the heterogeneity of rock considered by assuming that the mechanical properties conform to the Weibull distribution. Hu *et al.* (2014) simulated injection of water in a wellbore and hydraulic fracture at an arc dam heel. The crack was described by an equivalent anisotropic continuum with degraded material properties in the direction normal to the crack orientation.

The use of elasto-plastic constitutive models is mainly associated to cases of unconsolidated formations, where shear failure seems to be more important than tensile failure during the hydraulic fracturing process (Xu and Wong, 2010). The models used to adjust to this behaviour were Mohr-Coulomb and Drucker-Prager, associated with a tension cut-off. These can simulate not only fracture propagation due to plastification but also changes in effective stresses in the facture surroundings due to leak-off, which is high in this kind of permeable mediums. Busetti, Mish and Reches (2012) used an elasto–plastic damage model with pressure-dependent yielding, strain hardening and softening, and strain-based damage evolution to compare a four-point beam test, a dog-bone triaxial test and a hydraulic fracturing event with experimental results. The authors showed that the resolution of the damaged zone equivalent to a discrete fracture is determined by the coarseness of the FE mesh.

The use of enriched degrees of freedom to represent cracks explicitly has been gaining more and more focus by the academic community. Although this technique had been applied previously to mechanical cracks (Belytschko and Black, 1999; Dolbow, Moës and Belytschko, 2001) or simulation of flow in fractured medium (Réthoré, Borst and Abellan, 2006), the incorporation of the capability to propagate the hydraulic fracture has been developed more recently.

Mohammadnejad and Khoei (2013) and Khoei *et al.* (2014) developed a numerical model based on the extended Finite Element Method (XFEM) for the fully coupled hydro-mechanical analysis of deformable, progressively fracturing porous media interacting with the flow of two immiscible, compressible wetting and non-wetting pore fluids. The works point out that by allowing the interaction between various processes, that is, the solid skeleton deformation, the wetting and the non-wetting pore fluid flow and the cohesive crack propagation, the effect of the geomechanical discontinuity can be completely captured.

Zielonka *et al.* (2014) Compared analytical solutions (KGD and "Penny-Shaped") with interface elements and XFEM elements with cohesive behaviour in two and three dimensional models assuming toughness/storage dominated and viscosity/storage dominated propagation regimes, showing that both methods reproduce accurately the analytical solutions, and converge monotonically as the mesh is refined. Chen (2013) implemented the extended finite element method (XFEM) in Abaqus user subroutines for the solution of hydraulic fracture problems comparing the finite element results with the analytical solutions available in the literature.

Salimzadeh and Khalili (2015) used extended finite element method (XFEM) with the cohesive crack model as fracturing criterion to simulate hydraulic fracturing. Coupling between fracture and pore fluid was captured through a capillary pressure-saturation relationship, while the identical fluids in fracture and pore are coupled through a so-called leak-off mass transfer term. Model verification follows against analytical solutions available from literature. The authors showed that the results by single-phase flow might underestimate the leak-off.

Remij *et al.* (2015) presented an enhanced local pressure model for modelling fluid pressure driven fractures in porous saturated materials. The authors reconstructed the pressure gradient due to fluid leakage near the fracture surface based on Terzaghi's consolidation solution, ensuring that fluid flow happens exclusively in the fracture and that it is not necessary to use a dense mesh near the fracture to capture the pressure gradient.

Sobhaniaragh, Mansur and Peters (2016) presented a numerical technique based on the Cohesive Segments Method in combination with Phantom Node Method, called CPNM, to simulate 3D non-planar hydraulically driven fracture problem in a quasi-brittle shale medium. The authors used two different key scenarios, including sequentially and simultaneously multiple hydraulic, showing that later stages in sequentially hydraulic fracturing mainly secure larger values of fracture opening than what is achieved with simultaneously hydraulic fracturing. This effect can be attributed to the effect of stress interactions among fractures.

Mohammadnejad and Andrade (2016) modelled pump-in/shut-in tests in order to capture the bottom-hole pressure/time records and extract the confining stress perpendicular to the direction of the hydraulic fracture propagation from the fracture closure pressure.

Youn (2016) presented in his thesis the development and validation of an advanced hydro-mechanical coupled finite element program based on XFEM in order to estimate wellbore bottom-hole pressure during fracture propagation. The same research also considers material heterogeneity to check the effect of random formation property distributions on the hydraulic fracture geometry. The work uses a new stochastic approach combining XFEM and random field which is named eXtended Random Finite Element Method (XRFEM).

<u>**Boundary Element Method**</u>

In the reviewed works that use the Boundary Element Method (BEM), the method is often applied to simulate the rock mechanical behaviour, with subsequent coupling to the flow solution of a finite element or finite differences method. This method shows advantages because it only requires discretization along the fracture, demanding less CPU capacity.

Carter *et al.* (2000), Carter, Ingraffea and Engelder (2000), Sousa, Carter and Ingraffea (1993) use a linear elastic boundary element program with special hypersingular integration techniques and provide an elastic influence matrix that relates the unit pressures at the nodes on the crack surface with the elastic displacements. This matrix is then used along with the equilibrium fluid pressures to determine the overall structural response due to both the far field boundary conditions and the fluid pressure in the crack.

<u>Discrete Element Method</u>

Two types of models based on the Discrete Element Method (DEM) were found in the review. The Particle Flow Model and the Distinct Element Method.

The Particle Flow Model simulates the movement and interaction of circular particles. The constitutive behaviour of the rock is simulated by associating a contact model with each contact, as seen Figure 2.4 by Shimizu, Murata and Ishida (2011).

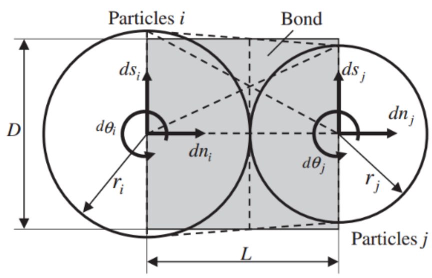


Figure 2.4 – Bonded particles model (Shimizu, Murata and Ishida, 2011)

Thus, HF can be modelled by assuming that a rock is made up of individual particles of specific stiffness bonded with bonds of specific strength. Shimizu, Murata and Ishida (2011) showed that under the applied load, the bonds between the particles can break, and a small crack can form. The crack pattern develops automatically with no need for remeshing. The calculation cycle is a time-stepping algorithm that requires the repeated application of the law of motion for each particle and a force-displacement law for each contact. The seepage effect can be modelled by adopting a fluid "domain" and fluid "pipe" (see Figure 2.5).

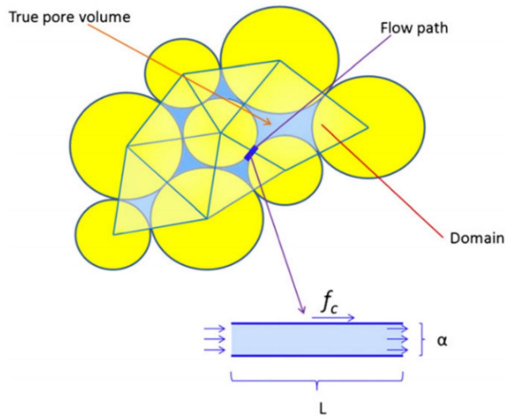


Figure 2.5 – Domains and flow paths in a bonded assembly of particles (Wang et al., 2014)

A "domain" is defined as a closed chain of particles, in which each link in the chain is a bonded contact. Each domain holds a pointer, via which all domains become connected. Meanwhile, a "pipe" is not only a fluid channel in a solid but also a channel connecting "domains", which is considered to be tangential to each bonded contact. The aperture of a "pipe" is proportional to the normal displacement of the contact. It changes when the contact breaks or when the particle moves. The volume of a "domain" is related to the number and aperture of the surrounding pipes. In addition, the water pressure in the "domain" continually changes as the coupling calculation proceeds, and it is applied to each particle as a body force with the flow in the channel being modelled using the Poiseuille equation (Wang *et al.*, 2014).

The second type of model based on DEM is the Distinct Element Method. It refers to the particular discrete-element scheme that uses deformable contacts and an explicit, time-domain solution of the original equations of motion (Nagel *et al.*, 2011). In this method, the rock mass assembly of deformable blocks interfaces a joint network which describes the interaction between distinct blocks. The deformation of each block is modelled by internal discretization. In Nagel *et al.* (2011) and Hamidi and Mortazavi (2014) the deformable rock blocks are modelled with the finite difference method. Considering the interaction of intact blocks and joints, DEM can effectively calculate the mechanical behaviour of block systems under different stress and displacement boundary conditions. Fracture propagation occurs in the bounds between blocks when the stress state reaches a certain limit value. Fluid flow occurs only in the joints and there is no porous flow in the rock matrix. Also, the numerical resolution of transient flow is done by using the finite difference scheme (Hamidi and Mortazavi, 2014).

Finite Differences Method

The Finite Difference Method (FDM) is commonly associated to other methods, such as the FEM. However, in the works reviewed, three applications use exclusively the FDM. Two of them considered a continuum three-dimensional medium with linear elastic (Zhou and Hou, 2013) or elasto-plastic (Agarwal and Sharma, 2011) behaviour, using Biot's theory to couple the mechanical and flow phenomena, in a similar way as some of the reviewed FEM models.

One additional model used a a simplified, and also computationally more efficient version of the particle flow model. A lattice, consisting of point masses (nodes) connected by springs, replaces the particles and contacts (respectively) of the particle flow model. Two springs that represent elasticity of the rock connect the lattice nodes, one representing the normal and the other shear contact stiffness. The solution of the equations of motion (three translations and three rotations) for all nodes in the model adopts a central difference scheme FDM. The relative displacements of the nodes are then used to calculate the force change in the springs. If the force exceeds the calibrated spring strength, the spring breaks and a micro crack forms. Fluid flow occurs through the network of pipes that connect fluid elements, located at the centres of either broken springs or springs that represent pre-existing joints. The model uses the lubrication equation to approximate the flow within a fracture (Damjanac *et al.*, 2013).

2.2. Intersection between hydraulic and natural fractures

2.2.1. Introduction

The behaviour of a hydraulic fracture near a natural fault or discontinuity is of great importance for an efficient reservoir simulation, as natural discontinuities can significantly influence the hydraulic fracturing process (Zhang and Ghassemi, 2011).

As proven in laboratory tests, a variety of events may happen when a hydraulic fracture intersects a natural fracture. Gu *et al.* (2012) gives a clear description of the different phenomena that occur:

1. First, the fracture tip reaches the interface (Figure 2.6a), but the fluid front remains behind because of the fluid lag. At this moment, the net fluid pressure (the difference between the fracturing-fluid pressure and the minimum in-situ stress) at the intersection point can be considered zero, but the natural fracture is already under the influence of the stress field generated by the hydraulic fracture. This step can be analysed by the mechanical interaction between the hydraulic fracture and the natural fracture without considering fluid flow. There are two possible outcomes from this interaction:
    - slippage or arrest (Figure 2.6b),
    - crossing (Figure 2.6c).
2. The second step in the process occurs soon thereafter when the fluid front reaches the natural fracture and fluid pressure at the intersection point rises.
    - In the case of slippage, the fluid may flow into the natural fracture and dilate it if the fluid pressure is larger than the normal compressive stress on the natural fracture. If flow continues, the dilated natural fracture becomes part of a hydraulic-fracture network (Figure 2.6d), i.e., the hydraulic fracture turns and propagates along the natural fracture.
    - Two possibilities exist for the case of crossing. In the first case, the natural fracture remains closed if the fluid pressure is less than the normal stress on the natural fracture (Figure 2.6e). In this case, the hydraulic fracture remains planar, and there may be enhanced leak-off if the filling material inside the natural fracture is permeable. The other possibility is that the fluid pressure is greater than the normal stress and the fluid flows into the opened natural fracture. In this case, the hydraulic fracture branches into the natural fracture, multiple fracture fronts propagate, and a complex network forms. As pumping continues, the fracture behaviour continues to evolve. For example, in the case of (Figure 2.6d), the hydraulic fracture may leave the path of the natural fracture and propagate again along the

preferred direction (perpendicular to the minimum horizontal stress). The hydraulic fracture may reinitiate at the intersection point (Figure 2.6f), at some weak points along the natural fracture, or at the end of the natural fracture. In the case of Figure 2.6e, the natural fracture may open later when the fluid pressure at the intersection rises further and overcomes the normal stress on the natural fracture.

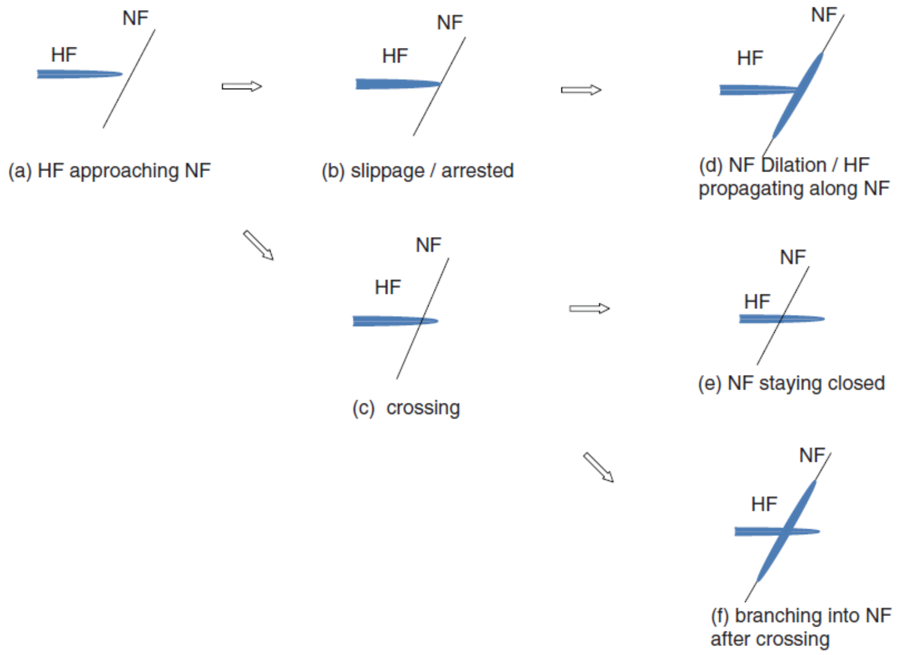


Figure 2.6 – Breakdown of the interaction process between hydraulic fracture (HF) and natural fracture (NF) (Gu et al., 2012)

Both Academia and the Industry put effort in understanding the mentioned phenomena by means of the monitoring of laboratory or field tests. The difficulties associated with the observation of such phenomena gives way to the numerical tools to work as a complement to increase the knowledge around the subject.

2.2.2. Field and laboratory tests

In the early days of research in this subject, Lamont and Jessen (1963) have tested 106 outcrop rock samples under triaxial stress conditions, showing that an existing fracture would have little effect on the hydraulic fracture. They also concluded that in every successful test there was fracture crossing. However, the authors assumed that around 30% of the tests were unsuccessful when the hydraulic fractures did not cross the existing fractures because of bleed-off of the fracture fluid at the top and bottom faces or at the ends of the existing fracture. In this author's opinion, this may lead to the assumption that in such cases the event of natural fracture opening occured. Lamont and Jessen (1963) also stated that the lower the angle between fractures, the further the path deviated from the centre line of the model. This deviation was always toward that part of the existing fracture which was closer to the injection end of the model, as Figure 2.7 shows.

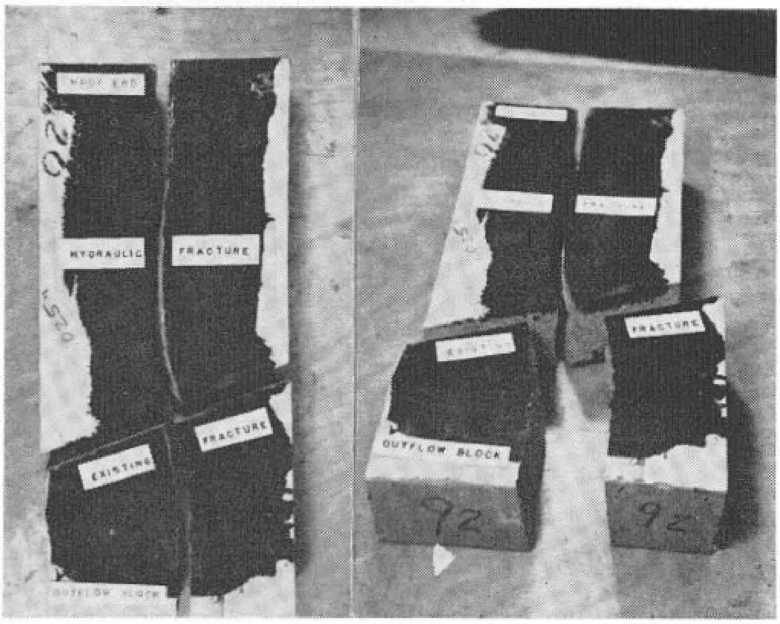


Figure 2.7 – Leuders Lime model with angle of bearing of 70° (Lamont and Jessen, 1963)

Daneshy (1974) performed experiments to study how hydraulic fractures evolved in the presence of natural flaws, observing that crossing occurs when the natural faults are closed and arrest happens in all other situations. Hanson, Shaffer and Anderson (1981) used small-scale laboratory experiments to study the effects of frictional characteristics on hydraulic fracture growth across unbounded interfaces in rocks, concluding that decreasing friction reduces the tendency of the crack crossing the interface.

Blanton (1982) executed laboratory tests on naturally fractured blocks of Devonian shale and hydrostone using different intersection angles under different triaxial states of stress. Figure 2.8 shows that the hydraulic fractures were mostly arrested by the natural fracture or opened the natural fracture, with exception of cases with high differential stresses and high angles of approach, where crossing occurred.

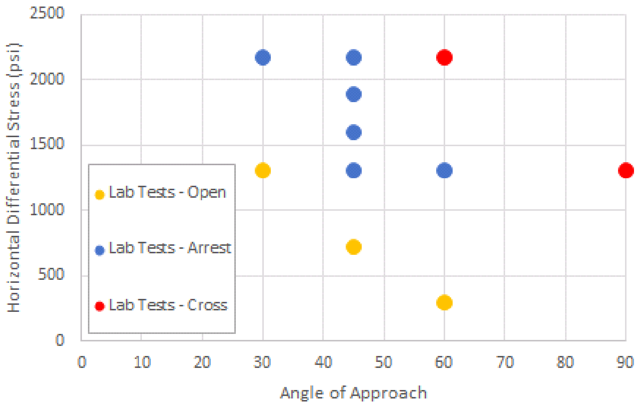


Figure 2.8 – Type of interaction observed at different combinations of differential stress and angle of approach (adapted from Blanton (1982))

Zhou and Xue (2011) carried out six tests on multiple naturally fractured blocks varying the in-situ stresses. Three types of fracture network patterns after propagation resulted. The authors showed that for high in-situ differential stresses the hydraulic fracture tends to dominate. As the differential stresses decrease, the hydraulic fracture propagates with branches. For extreme low differential stresses, natural fractures tend to dominate fracture geometry.

Gu *et al.* (2012) conducted six tests on sandstone samples with varying fracture angles and initial confining stresses, showing that the fracture is more likely to turn and propagate along the interface than to cross it when the angle is less than 90º. Cheng *et al.* (2014) performed 24 tests on cement blocks with variation of confining stresses and three-dimensional angle between fractures (dip and strike angles). The results showed that crossing happens in models with high approaching angles and high horizontal stress differences. The knowledge accumulated by the mentioned tests allowed the authors to make predictions and further comparisons with field microseismic results in a real case study. The same authors also showed that above a critical pump displacement or above a critical viscosity, the hydraulic fracture tends to cross the natural fracture. On the other hand, below the critical values hydraulic fracture propagates along the natural fracture rather than crossing it (Cheng, Jin, Y. Chen, *et al.*, 2014)

Khoei *et al.* (2015) carried out hydraulic fracturing experimental tests in fractured media under plane strain conditions, with the experimental tests being continued until the hydro-fracture merged with the natural fault. A number of tentative experiments showed that the intersection of hydro-fracture with the natural fault is characterized by an abrupt loss of the water level in the pump fluid tank

As expected, due to the complexity involved, the number of field tests found in the literature is small. Murphy and Fehler (1986) used microseismic observations to claim that the shear slippage along the natural discontinuities can be activated before the conventional tensile failure occurs, especially in the presence of high differences between the minimum and maximum in-situ stresses. Based on their observations, the occurrence of slippage along the natural fracture faces leads to the hydro-fracture branching, or dendritic evolution patterns, which are in agreement with microearthquake locations.

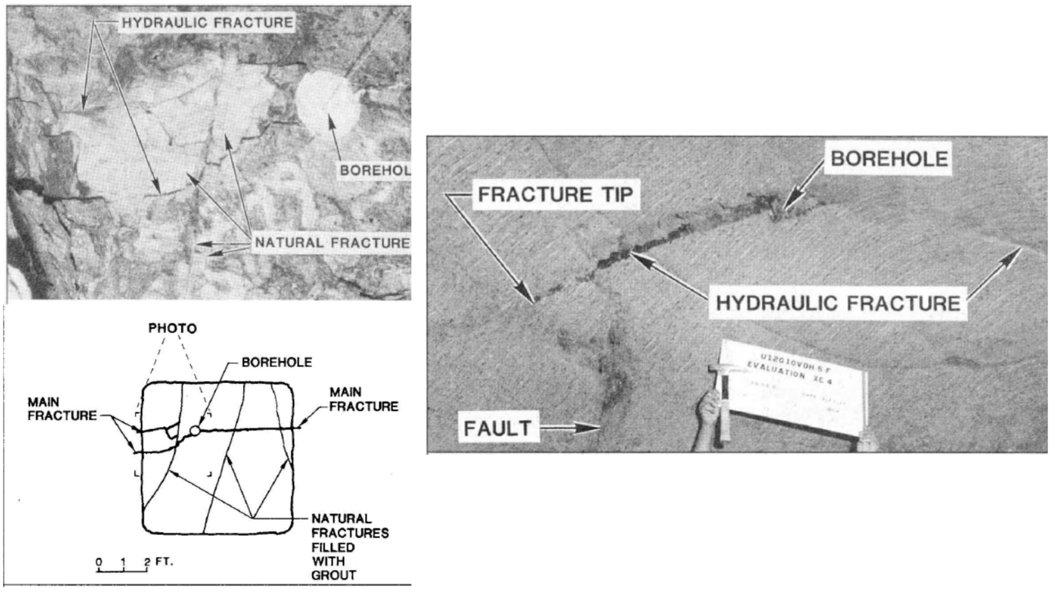


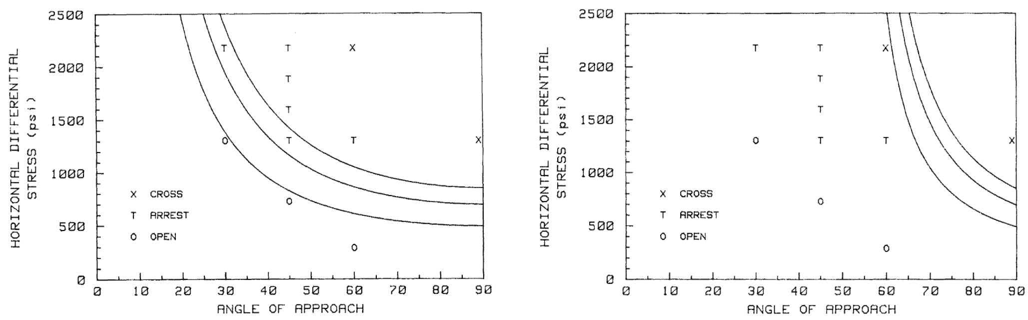
Figure 2.9 – Pictures from the mineback observations (Warpinski and Teufel, 1987)

Warpinski and Teufel (1987) presented perhaps the only field study with large-scale and direct observations in the literature. The authors integrated results from mineback experiments (425 m depth) with laboratory experiments to explain the influence of geologic discontinuities in hydraulic fracturing. Figure 2.9 presents some pictures of the mineback work. This study concluded that geologic discontinuities may influence fracture height, length, leak-off, treatment pressure, and proppant transport. The effect of the discontinuities depends on many parameters, such as the permeability of the joints, frictional properties, in-situ stresses, joint spacing and orientation, treatment pressure, and fracture fluid leak-off viscosity.

2.2.3. Analytical models

In some of the research works mentioned in Section 2.2.2, the field tests gave empirical support to analytical methods developed by the authors. These methods mainly focus on predicting the intersection behaviour. Most of these criteria depend on the differential in-situ stress, angle of approach, friction in the natural fracture, rock tensile strength and fracture energy.

Blanton (1982) used an equation to compute the fracture stress state, and then define which type of intersection occurs by comparing the stress state with the pressure applied by the fluid. Figure 2.10 shows plotting of the analytical solutions (for different fracture energies) against the laboratory tests.

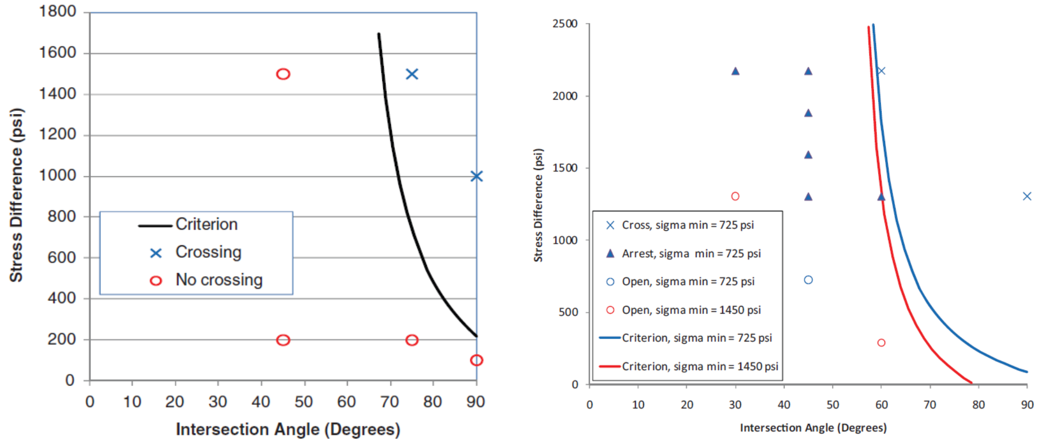


a) b)

Figure 2.10 – Comparison of laboratory tests with analytical criteria. a) Opening criterion. b) Arresting criterion (Blanton, 1982)

Zhou *et al.* (2008) studied the hydraulic fracture propagation behaviour in naturally fractured reservoirs through a series of triaxial fracturing experiments, operating different values of horizontal stress, angle of approach, and shear strength of pre-existing fracture. The authors observed two hydraulic fracture patterns in different stress regimes. In a normal stress regime, it leads to fractures, with interacting branches because of the pre-existing fracture. Tortuous fractures were found along the fracture height when one of the horizontal stresses is the maximum principle stress.

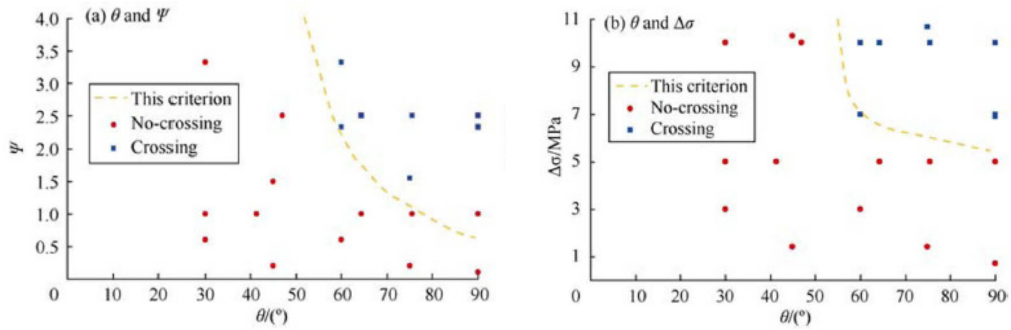
Gu *et al.* (2012) have developed a criterion to determine if a fracture crosses a frictional pre-existing interface at non-orthogonal angles, validating it with laboratory tests as stated previously. This criterion is an extension of Renshaw and Pollard (1995) for orthogonal intersections of fractures with material interfaces. Figure 2.11 shows the results obtained in the reference. .



a) b)

Figure 2.11 – Comparison of laboratory tests with Gu's analytical criterion. a) Gu's tests. b) Blanton's tests

Cheng *et al.* (2014) developed a three-dimensional analytical model to predict crossing which assumed that crossing occurs when two conditions are met: first, the maximum tensile stress at the hydraulic fracture tip is equal to the tensile strength of the rock on the opposite side of the natural fracture; second, no shear slippage occurs on the natural fracture surface. Results of Figure 2.12 show that the criterion fits very well to the laboratory tests for the relations between dip angle, strike angle and differential confining stress.



a) b)

Figure 2.12 – Comparison of laboratory tests with Cheng's analytical criterion. a) dip vs strike angles space b) dip vs differential stresses space (Cheng *et al.* (2014))

2.2.4. Numerical models

Despite the advances in modelling with numerical tools, most models in the literature still assume that the hydraulic fracture is a single planar fracture. This contrasts with the fact that multistranded hydraulic-fracture geometry is a common occurrence (Dahi-Taleghani and Olson, 2011; Zhang and Ghassemi, 2011). Consequently, single-crack models may result in loss of accuracy if fracture interaction with natural fractures is not taken into account.

In the past few years, researchers focused more on this specific subject, resulting in developments in understanding how natural fractures affect a hydraulic fracturing treatment. Similarly to the studies described in Chapter 2.1.3.2, different techniques were used to simulate numerically the interaction between hydraulic and natural fractures.

Finite Element Method

Dyskin and Caballero (2009) investigated the interaction between the hydraulically driven fracture and frictionless natural fault using the finite element method, and illustrated that a relatively long frictionless and cohesionless fault is capable of arresting the hydraulic fracture propagation.

Dahi-Taleghani and Olson (2011) presented a numerical model based on enriched nodes to study fracture intersections by tracking fluid fronts in the network of reactivated fissures, where the hydraulic fracture was arrested by pre-existing natural fractures, and/or was controlled by shear strength and potential slippage at the fracture intersections. The same authors performed analyses in full scale fractured reservoirs (see Figure 2.13) and showed that when natural fractures are perpendicular to the direction of the hydraulic fracture, the largest possible debonded zone may form, which is equivalent to the optimum case to stimulate a reservoir.

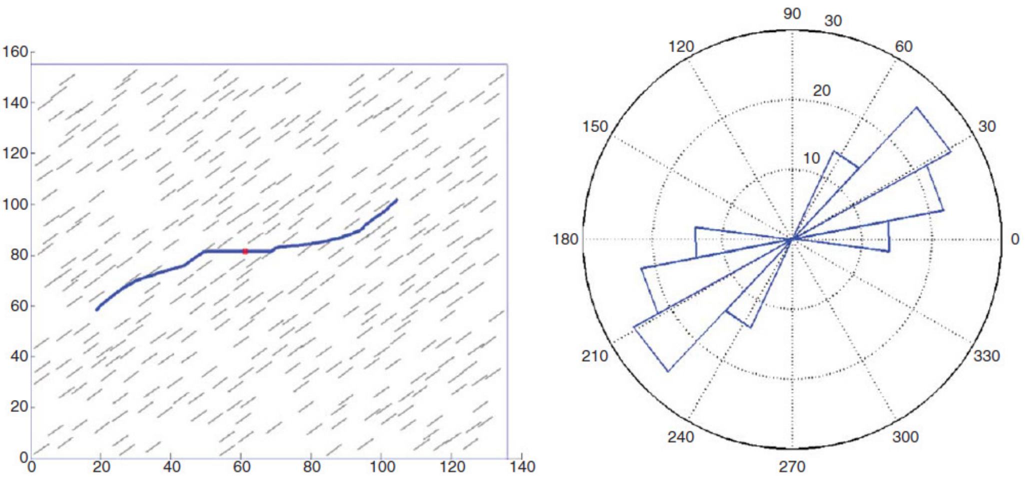


Figure 2.13 – Resultant hydraulic fracture pattern and rose diagram in the case where natural fractures make a 45° angle with the original orientation of the hydraulic fracture (Dahi-Taleghani and Olson, 2011)

Zhang and Ghassemi (2011) performed a comprehensive study on the interaction between the hydraulic fracture and natural fault, and concluded that the fault influence is conditioned by its shear stiffness, its inclination, and its distance from the hydraulic fracture. It was also highlighted that the hydraulic fracture always tends to propagate along the maximum compressive stress direction.

Keshavarzi, Mohammadi and Bayesteh (2012) studied the interaction between hydraulic and natural fractures using the XFEM, considering a constant and uniform net pressure throughout the hydraulic fracture system. They compared numerical simulations with the laboratory tests of Blanton (1982) and showed that natural fractures most probably divert hydraulic fractures at low angles of approach while at high horizontal differential stress and angles of approach of 60 or greater, the hydraulic fracture crosses the natural fracture. Keshavarzi and Jahanbakhshi (2013) compared the XFEM results of fracture interactions studies (see Figure 2.14) with a neural network that was developed based on horizontal differential stress, angle of approach, interfacial coefficient of friction, Young's modulus of the rock and flow rate of the fracturing fluid. The results indicated that the developed Artificial Neural Network was not only feasible but also yields quite accurate outcome.

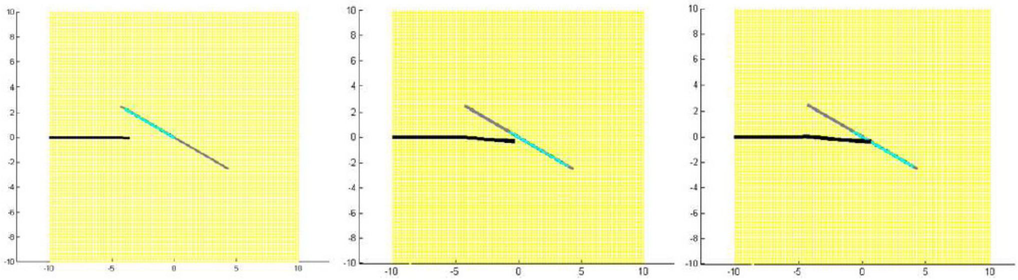


Figure 2.14 – Hydraulic fracture and natural fracture behaviour as hydraulic fracture is propagating toward the pre-existing natural fracture and intersects with it. Light blue represents the debonded zone of the natural fracture (Keshavarzi, Mohammadi and Bayesteh, 2012)

Khoei, Vahab and Hirmand (2016) modelled the interaction between the fluid-driven fracture and frictional natural fault using an enriched-FEM technique based on the partition of unity method. The intersection between two discontinuities was modelled by introducing a junction enrichment function. The medium is considered impermeable and the fluid pressure within the fracture was assumed constant throughout the propagation process. The frictional contact behaviour along the fault faces was modelled using an X-FEM penalty method. The authors showed that a lower value of fault length together with a larger frictional resistance along the natural fault produces a larger vertical tensile stress ahead of the intersection point of two discontinuities, and increases the possibility of penetration of the hydro-fracture through the natural fault. One further conclusion of the work is that the far-field stress conditions have a significant effect on the performance of internal pressure imposed on the hydro-fracture faces, and plays an important role on the mechanism of interaction between the hydro-fracture and natural fault. Moreover, it was concluded that there is a wide range of parameters that may affect the overall behaviour of the interaction mechanism, including the hydraulic fracture/natural fault configuration, the fault inclination angle, far-field stress conditions, and the frictional resistance along the natural fault.

In Khoei *et al.* (2015) the results of two hydraulic fracturing experimental tests performed on impermeable rock blocks with natural discontinuities were compared with those obtained from the X-FEM numerical model, showing very good agreement between the numerical and experimental results. It was shown that the shear strength of the natural fault plays a key role in the mechanism of interaction, including the arrest, penetration, offset crack propagation, and diversion when the hydro-fracture merges with the natural fault.

Other Methods

Dong and De Pater (2001) used the boundary element method for the simulation of hydraulic fracturing and its interaction with faults. The work was based on the displacement discontinuity method, which was first presented by Crouch and Starfield (1983), and concluded that a fault has an evident effect on the crack propagation.

Zhang and Jeffrey (2006) modelled a fluid-driven fracture intersecting a pre-existing fracture using the displacement discontinuity method and the finite difference method to deal with the coupling mechanism of rock fracture and fluid flow. It was stated that in the presence of pre-existing fractures, the fluid-driven cracks can be arrested or retarded in growth rate as a result of diversion of fluid flow into and frictional sliding along the pre-existing fractures. Frictional behaviour significantly affects the ability of the fluid to enter or penetrate the pre-existing fracture only for those situations where the fluid front is within a certain distance from the intersecting point. The authors also showed that fracture re-initiation from secondary flaws can reduce the injection pressure, but re-initiation is suppressed by large sliding on pre-existing fractures that are frictionally weak.

Nagel *et al.* (2011) used the Distinct Element Method to model discontinuities governed by Mohr Coulomb as boundary interactions between blocks. The deformable blocks were subdivided into a mesh of finite differences elements and the flow model included a system of flow planes. The simulation of injected well with natural fractures was performed and the fracture geometry was defined by means of a Discrete Fracture Network (DFN), as shown in Figure 2.15.

Kresse *et al.* (2014) proposed a tool that, although based on very simple methods, gathered many phenomena that affect hydraulic fracture propagation in fractured reservoirs. The coupled fluid flow and elastic deformation equations were defined with similar assumptions of conventional pseudo-3D fracture models and the stress effects between fractures given by Theory of Dislocations. The interaction with natural fractures is based on an analytical crossing model and the fracture geometry is defined in an unconventional fracture model (UFM), as shown in Figure 2.16. The implemented model solves a system of equations governing fracture deformation, height growth, fluid flow, and proppant transport in a complex fracture network with multiple propagating fracture tips. Simulation results from the model showed that stress anisotropy, natural fractures, and interfacial friction play critical roles in creating fracture network complexity.

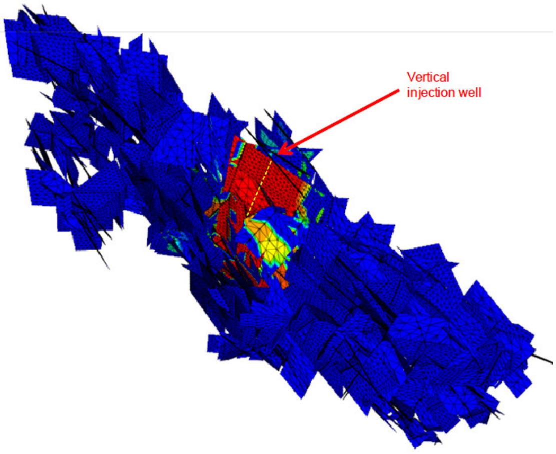


Figure 2.15 – Pore pressures in the model (Nagel et al., 2011)

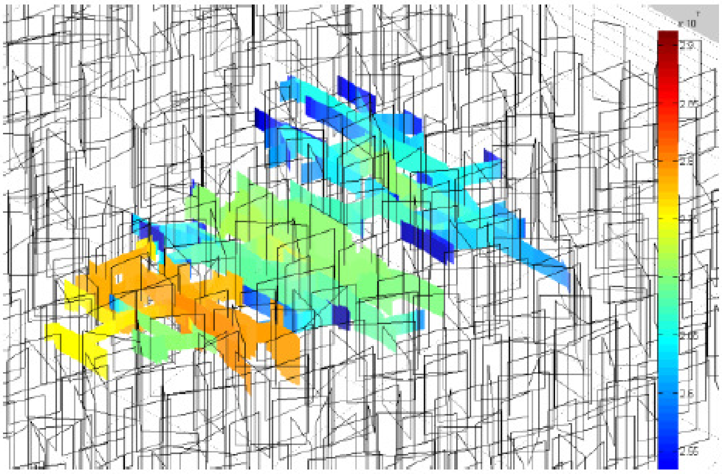


Figure 2.16 – Fluid pressures in the fracture network (Kresse et al., 2014)

Damjanac *et al.* (2013) presented a code that uses a three-dimensional lattice representation of brittle rock consisting of point masses (nodes) connected by springs with the pre-existing joints being derived from a user-specified discrete fracture network (DFN). Non-steady, hydro-mechanically coupled fluid flow and pressure within the network of joint segments and the rock matrix were also considered. The equation of motion is solved for all lattice nodes using the Finite Difference Method. The springs between the nodes break when their strength (in tension) is exceeded, corresponding to the formation of microcracks, which link to form macro scale fractures. The authors applied the proposed code to the simulation of an injected well with natural fractures, as Figure 2.17 shows.

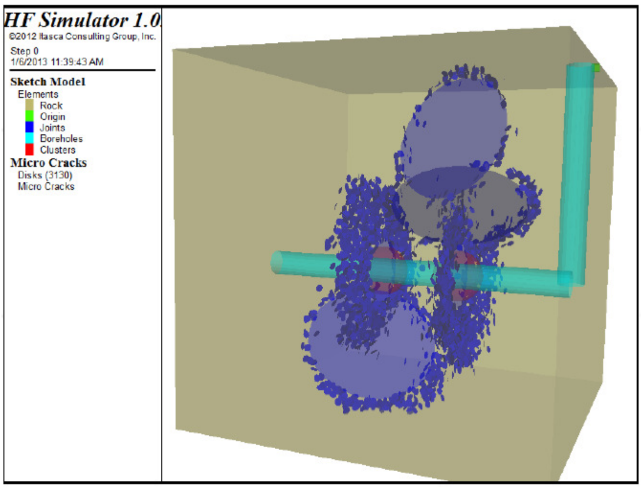


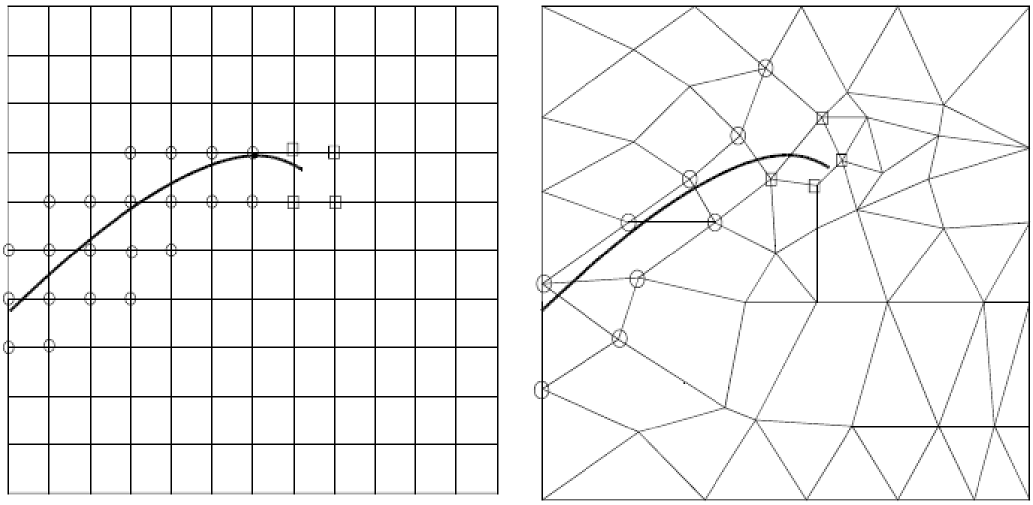
Figure 2.17 – Hydraulic fractures generated in a medium with three pre-existing joints (blue disks are microcracks) (Damjanac et al., 2013)

2.3. The eXtended Finite Element Method

2.3.1. Introduction

The eXtended finite element method (XFEM) is a technique to model strong (displacement) or weak (strain) discontinuities over a conventional finite element model. This technique was first presented by Belytschko and Black (1999), following research on enrichment strategies presented Benzley (1974). It was presented as a minimal remeshing finite element method for crack growth based on setting special enrichment functions to extra degrees of freedom along the fracture tip to capture the field singularities. The authors supported the method in the partition of unity property, presented by Melenk and Babuska (1996), which basically states that the shape functions in any point inside a finite element may be affected of local approximation functions, as its sum is kept equal to one.

Moes and Dolbow (1999) developed the method in order to avoid any type of remeshing, by using the Haar function in the fracture body and tip functions in the fracture tip. Figure 2.18 shows the nodes that are affected by the method, where the circled nodes represent the fracture body and the squared nodes the fracture tip. The method treats the crack as a completely separate geometric entity and the only interaction with the mesh occurs in the selection of the enriched nodes. The authors highlight how accurately the stress intensity factors can be computed with relatively coarse meshes and how it is readily generalized to other problems such as those in three dimensions and involving nonlinear materials. As the main drawback, it is pointed out that there is the need to account for a variable number of degrees of freedom per node.



a) b)

Figure 2.18 – Discontinuity on a structured mesh (a) and on an unstructured mesh (b). The circled nodes are enriched by the jump function whereas the squared nodes are enriched by the branch tip functions (Moës and Belytschko, 2002)

Sukumar *et al.* (2000) scaled the XFEM implementation for three-dimensional problems, comparing the results with penny and elliptical analytical solutions and showing that a good agreement was obtained. Wells and Sluys (2001) and Moës and Belytschko (2002) extended the implementation to quasi-brittle materials, by considering a cohesive zone at the crack tip, showing the effectiveness of the proposed method through simulations of cohesive crack growth in concrete.

The use of the XFEM in quadratic elements was presented by Stazi *et al.* (2003). Lee *et al.* (2004) combined a mesh superposition method with the XFEM to model stationary and growing fractures. The fracture tip field was modelled by superimposed quarter point elements on an overlaid mesh, and the body of the discontinuity was implicitly described by a step function on partition of unity.

Khoei (2008) presents in his book an extensive overview about the theoretical and practical application of the XFEM in continuum mechanics.

Similarly to the present research, different authors have implemented the XFEM using commercial software, such as Abaqus. Giner *et al.* (2009) and Silva (2015) implemented an XFEM element using Abaqus UEL user subroutine to simulate mechanical problems based in linear elastic fracture mechanics and non-linear frictional contact analyses. Chen (2013) also considered fluid pressure degrees of freedom to describe the fluid flow within the crack and its contribution to the crack deformation, thus modelling hydraulic fracture problems.

2.3.2. Fracture geometry in XFEM

In the XFEM, the fracture geometry is independent of the mesh and its presence is taken into account by creating enrichment degrees of freedom and applying local functions to those. In order to correctly and efficiently represent the fracture geometry, different techniques were used.

The level set function (LSF), by Osher and Sethian (1988) is the most frequently used technique with the XFEM to implicitly define the location and geometry of a discontinuity. Basically, two functions are used to represent the fracture at any point of the domain, one for the crack body and the other for the crack tip. Then, the values of the enrichment functions at any degree of freedom may be taken from the LSF, directly (signed distance) or indirectly (tip enrichment functions) (Fries and Baydoun, 2012). This technique may also be used for crack growth as new segments update the LSF when propagation occurs.

More advanced LSF techniques where developed later, such as Ventura, Budyn and Belytschko (2003) who introduced the LSF consisting of vectors to describe a propagating fracture in the element-free Galerkin method. Ji, Chopp and Dolbow (2002) presented a hybrid XFEM-LSF to model the evolution of fluid phase interfaces to represent temperature jump.

Sukumar *et al.* (2008) solved three-dimensional problems by combining the XFEM with the fast marching method, which was originally developed by Sethian (1996) and is characterized by avoiding the need to represent the geometry of the interface during its evolution by tracking the first arrival of the interface as it passes a point.

2.3.3. XFEM with coupled problems

The flow of fluids in deformable porous media has been studied via the XFEM framework to analyse the physical behaviour of many issues in geotechnical and petroleum engineering (Youn, 2016). De Borst, Réthoré and Abellan (2006) analysed a two phase fluid saturated media for a biaxial plane strain case with a discontinuity propagation. Réthoré, de Borst and Abellan (2006) presented a two-scale approach of the XFEM for fluid flow within a deforming unsaturated and progressively fracturing porous medium and Réthoré, de Borst and Abellan (2007) modelled dynamic shear band propagation in a fluid-saturated medium. Gracie and Craig (2010) applied the XFEM for predicting the steady state leakage from layered sedimentary aquifer systems perforated by abandoned wells, showing that for coarse meshes this technique proved to be more than two orders of magnitude more accurate than the standard FEM. Huang *et al.* (2011) proposed an enrichment scheme to compute model fractures and other conduits in porous media flow problems that could capture effects of local heterogeneities introduced by subsurface features of the pressure solution.

Silvestre *et al.* (2015) implemented an enriched element to compare the coupled behaviour of fractured materials with analytical solutions and with examples simulated in other software. Lamb, Gorman and Elsworth (2013) presented a fracture mapping approach combined with the extended finite element method to simulate coupled deformation and fluid flow in fractured porous media using a transfer function to model the flow interaction between the porous matrix and existing fractures. Sheng *et al.* (2015) (see Figure 2.19) presented a numerical framework to simulate coupled deformation and fluid flow in porous media, also addressing problems with arbitrary orientation and intersection of sealed fractures.

As the modelling of hydraulic fractures is also a coupled problem, many other research works in this area are presented in Chapters 2.1.3 and 2.2.4.

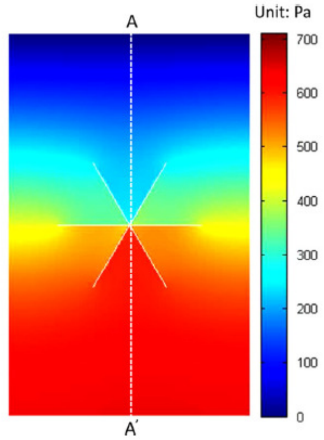


Figure 2.19 – Excess pore pressure field (Sheng et al., 2015)

2.3.4. XFEM with fracture branching or crossing

The consideration of multiple fractures that intersect each other within the XFEM concept was introduced by Daux, Moes and Dolbow (2000), through the concept of an enriched junction function to be used at each intersection. Budyn *et al.* (2004) applied the XFEM technique for multiple fractures growing and interacting within both homogeneous and inhomogeneous brittle materials. Zi *et al.* (2004) provided an approach to model multiple fracture propagation and intersection in a quasi-brittle cell with random minor fractures.

Duarte, Reno and Simone (2007) presented high-order implementations of a generalized finite element method for three-dimensional branched cracks (see Figure 2.20) showing that convergence rates obtained are close to those of problems with smooth solutions.

The same methodology was used by Chen and Lin (2010) to compute the T-Stress in the branch crack problem and Das, Sandha and Narang (2013) to study the behaviour of rock bolts for improvement in ground support.

Siavelis *et al.* (2013) used junction functions to simulate large sliding along branched discontinuities, running several examples, including a 3D geological graben with branching faults.

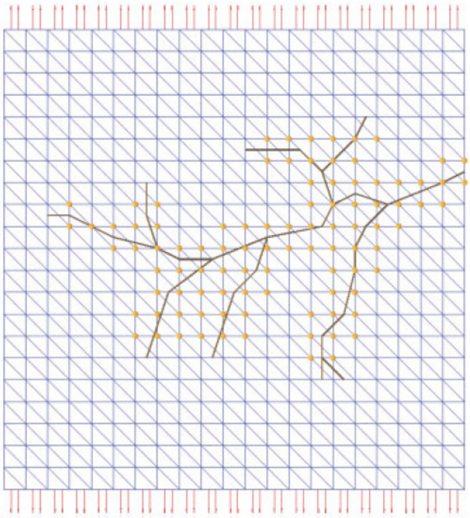


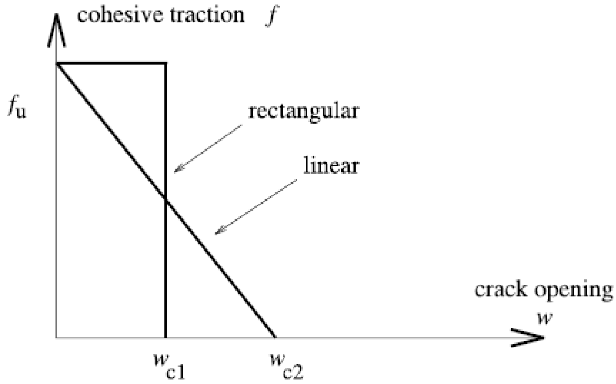
Figure 2.20 – Enriched nodes represented by circles (Duarte, Reno and Simone, 2007)

2.3.5. Crack tip behaviour in XFEM

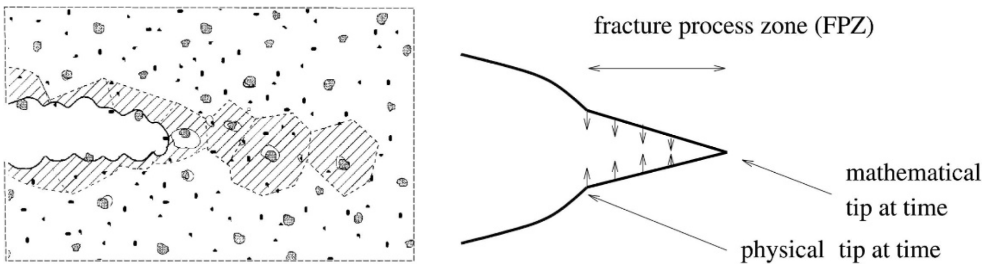
In the early years of research with XFEM, most academic works (Belytschko and Black, 1999; Sukumar *et al.*, 2000; Belytschko *et al.*, 2001; Ventura, Budyn and Belytschko, 2003; Zi *et al.*, 2004) considered the tip behaviour by using a specific enrichment function based on an asymptotic stress field, following the Linear Elastic Fracture Mechanics (LEFM). Figure 2.18 shows that the tip nodes are considered only near the fracture tip. The asymptotic functions were based in sinusoidal functions and allowed to use propagation criteria based on stress intensity factors. Basically, a new fracture segment is created when the stress intensity factors at fracture tip are reached.

Aware of the relevance of the small-scale processes that occur at the fracture tip, which control the global response of the fracture, and of the complexity involved in constructing solutions for fluid driven factors (Detournay, 2004), Lecampion (2009) presented an XFEM formulation for the solution of hydraulic fracture problems by introducing special tip functions encapsulating tip asymptotic functions that represent the different regimes typically encountered in hydraulic fractures.

However, LEFM is only applicable when the size of the fracture process zone (FPZ) at the crack tip is small compared to the size of the crack and the size of the specimen (Bazant and Planes, 1998). In order to extend the use of XFEM to quasi-brittle materials, Wells and Sluys (2001) and Moës and Belytschko (2002) applied the cohesive crack concept, where the propagation is governed by a traction–displacement relation (see Figure 2.21a) across the crack faces near the tip. This behaviour is assigned to the region between the real physical tip and the mathematical tip, where the process zone ends (see Figure 2.21b). Moës and Belytschko (2002) considered that, since the stresses at the tip are not singular, non-asymptotic functions should be used for tip enrichment. Other authors used enriched techniques to simulate cohesive crack growth and showed it applicability to problems such as Mode I and Mixed Mode experimental tests (Mariani and Perego, 2003; Cox, 2009) or three and four point beam bending tests (Mergheim, Kuhl and Steinmann, 2005).



a)



b)

Figure 2.21 – Modelling of the fracture process zone. (a) Two cohesive laws with the same cohesive strength and fracture energy. (b) The extent of the cohesive zone at a certain moment (Moës and Belytschko, 2002; Wang, 2016)

Wells and Sluys (2001) modelled cohesive crack growth by considering only the jump function to represent the fracture and guaranteeing the closure of the tip by deactivating the jump enhancement at the nodes closest to the tip. This facilitates implementations, as only one enrichment function is required and concerns are avoided, such as the existence of blending elements. However, it must be stated that the fracture tip cannot lie inside one element, but only on its borders. Therefore, a propagation segment must always cross the element totally. Other research works were developed under this premise (Zi and Belytschko, 2003; de Borst, Remmers and Needleman, 2006; Comi and Mariani, 2007; Mougaard, Poulsen and Nielsen, 2007), as well as commercial software (Simulia, 2014).

2.3.6. Contact problems in XFEM

To simulate situations where compressive stresses lead to contact between fracture faces, different types of contact models have been implemented. The literature review identifies the most frequently used contact models in association with XFEM simulations are:

- Penalty Method;
- Lagrange Multipliers;
- Augmented Lagrange Multipliers;
- LATIN method.

The penalty method consists in using a high stiffness (penalty coefficient) between the fracture faces, when the faces are in contact. This way, when under compressive contact, two fracture faces suffer a slight overlap and the stresses obtained from that relative displacement are the normal contact stresses. This method is easy to implemented which does not require the introduction of constraints or degrees of freedom to represent contact. It also does not require the introduction of outer iterative loops for constraint check. On the other hand, the accuracy of satisfying equilibrium highly and ovelapping restrictions depends on the magnitude of penalty parameter. The larger the value of the penalty parameter, the more accurate is the solution. However, very large values for the penalty parameter may result in an ill conditioned formulation when the penalty parameter is combined with finite stiffness of bodies in contact. As stated by Grazina (2009), the process may intensify instability problems for paths that impose randomness in the relative displacements evolution. Khoei and Nikbakht (2006) and Liu and Borja (2008) applied this method to simulate frictional contact using standard Coulomb friction. More recently, Khoei and Mousavi (2010) presented a node-to-node contact algorithm for XFEM to model the large deformation-large sliding contact problem using the penalty approach.

The Lagrange Multipliers Method, considers extra degrees of freedom so the contact forces are computed as primary unknowns. The restriction of null relative displacement of the faces in contact is enforced exactly. The major limitation of this method is that it requires extra variables in the model, affecting the dimension and sparsity of the system of equations. According to Khoei (2008), other limitations may exists, such as the existence of diagonal values that take the value zero, leading to difficulties in finding a solution. Nistor *et al.* (2009) coupled the X-FEM with the Lagrangian large sliding frictionless contact algorithm while Siavelis *et al.* (2013) applied the same technique to three-dimensional problems where fractures intersect and branch.

The Augmented Lagrangian Method eliminates the drawbacks of penalty and Lagrange multipliers techniques, and attempts to achieve a predetermined tolerance for the contact constraint through an iterative procedure. The main idea of this technique is to combine the penalty and Lagrange multipliers methods to inherit the advantages of both techniques, that is, decreasing the ill-conditioning of governing equations, and essentially satisfying the contact constraints with finite values of penalty parameters (Khoei, 2008). The values of the penalty parameter are calculated iteratively in an outer loop until a predetermined tolerance is achieved and then, the non-linear FEM problem is solved in an inner loop. Elguedj, Gravouil and Combescure (2007) present an augmented Lagrangian formulation in the XFEM framework that is able to deal with elasto–plastic fatigue crack growth. Hirmand, Vahab and Khoei (2015) implemented this method using a return mapping algorithm for the Coulomb friction rule, showing good accuracy of the proposed model in simulations of straight, curved and wave-shaped discontinuities.

The LArge Time INcrement (LATIN) Method shares similar features with the Augmented Lagrangian Method, i.e. runs two iterative procedures, one for the convergence in the penalty constraint, and other for the non-linear system of equations. The two iterative procedures are solved separately until convergence is achieved in both, as Figure 2.22 shows. Dolbow, Moës and Belytschko (2001) were the first to incorporate contact and friction in crack faces with the XFEM to simulate crack growth under opening/closing modes using the LATIN Method. Gravouil, Pierres and Baietto (2011) scaled the same method to three dimensional models under cyclic fretting loading.

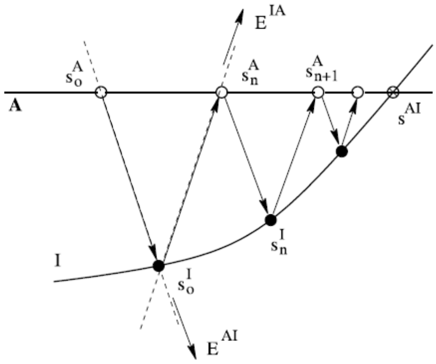


Figure 2.22 – The iterative procedure in the LATIN algorithm (Dolbow, Moës and Belytschko, 2001)