4.

Applications to single bridge decks

4.1. Introduction

In order to obtain a time-domain modeling of bridge deck flutter, the frequency dependent aerodynamics self-excited forces were approximated in the Laplace domain by rational functions. A matrix formulation of the rational functions using Karpel's "minimum state" was applied to aerodynamic data obtained for various bridge decks.

In the example presented below, the critical velocity of a bridge with a 2000m span and designed with an aerodynamic cross section that may be considered as having the same behavior of a flat plate when subjected to a wind stream is determined. The flutter derivatives were computed through the theoretical formulation of Theodorsen [84] in Chapter 2. The approximation functions were calculated in Chapter 3.

4.2. Calculation of the critical velocity of a bridge deck

One starts with the state-space system equations:

$$\begin{bmatrix} \ddot{\mathbf{q}} \\ \dot{\mathbf{q}} \\ \dot{\mathbf{x}}_a \end{bmatrix} = \mathbf{A} \, . \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{q} \\ \mathbf{x}_a \end{bmatrix}$$

where $\mathbf{A}$ is the state-space system matrix

$$\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}\left[\mathbf{C} - (\frac{B}{U})\mathbf{V}_f\mathbf{A}_1\right] & -\mathbf{M}^{-1}[\mathbf{K} - \mathbf{V}_f\mathbf{A}_0] & \mathbf{M}^{-1}\mathbf{V}_f\mathbf{D} \\ \mathbf{I} & 0 & 0 \\ 0 & (\frac{U}{B})\mathbf{E} & -(\frac{U}{B})\mathbf{R} \end{bmatrix} \qquad (4.1)$$

A is variable according to the wind velocity. The various terms of **A** read:

$$\mathbf{q} = \begin{bmatrix} h/B \\ \alpha \end{bmatrix} \tag{2.47}$$

$$\mathbf{V}_f = \begin{bmatrix} -1/2\,\rho B & 0 \\ 0 & 1/2\,\rho B^2 \end{bmatrix} \tag{2.48}$$

$$\mathbf{M} = \begin{bmatrix} mB & 0 \\ 0 & I_\alpha \end{bmatrix} \;;\; \mathbf{C} = \begin{bmatrix} c_h B & 0 \\ 0 & c_\alpha \end{bmatrix} \;;\; \mathbf{K} = \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix} \tag{2.49}$$

$$-\mathbf{M}^{-1}\left[\mathbf{C} - (\tfrac{B}{U})\mathbf{V}_f\mathbf{A}_1\right] = -\begin{bmatrix} mB & 0 \\ 0 & I_\alpha \end{bmatrix}^{-1} \left\{ \begin{bmatrix} c_h B & 0 \\ 0 & c_\alpha \end{bmatrix} - (\tfrac{B}{U}) \begin{bmatrix} -1/2\,\rho B & 0 \\ 0 & 1/2\,\rho B^2 \end{bmatrix} \begin{bmatrix} A_{1_{11}} & A_{1_{12}} \\ A_{1_{21}} & A_{1_{22}} \end{bmatrix} \right\} \tag{4.2}$$

$$-\mathbf{M}^{-1}[\mathbf{K} - \mathbf{V}_f\mathbf{A}_0] = -\begin{bmatrix} mB & 0 \\ 0 & I_\alpha \end{bmatrix}^{-1} \left\{ \begin{bmatrix} k_h B & 0 \\ 0 & k_\alpha \end{bmatrix} - \begin{bmatrix} -1/2\,\rho B & 0 \\ 0 & 1/2\,\rho B^2 \end{bmatrix} \begin{bmatrix} A_{0_{11}} & A_{0_{12}} \\ A_{0_{21}} & A_{0_{22}} \end{bmatrix} \right\} \tag{4.3}$$

$$\mathbf{M}^{-1}\mathbf{V}_f\mathbf{D} = -\begin{bmatrix} mB & 0 \\ 0 & I_\alpha \end{bmatrix}^{-1} \begin{bmatrix} -1/2\,\rho B & 0 \\ 0 & 1/2\,\rho B^2 \end{bmatrix} \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \tag{4.4}$$

$$\frac{U}{B}\mathbf{E} = \frac{U}{B}\begin{bmatrix} E_{11} & E_{11} \\ E_{11} & E_{11} \end{bmatrix} \tag{4.5}$$

$$-\frac{U}{B}\mathbf{R} = -\frac{U}{B}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{4.6}$$

The equations above are written for 2 lag terms, i.e., n_L=2. For n_L>2,

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \\ - & - \\ - & - \\ E_{n_L 1} & E_{n_L 2} \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} D_{11} D_{12} -- D_{1n_L} \\ D_{21} D_{22} -- D_{2n_L} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{22} & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & \lambda_{n_L n_L} \end{bmatrix}$$

where m is the mass of the bridge deck per meter and I_α the torsional mass moment of inertia per meter, B is the deck width and $\mu = m/\rho B^2$ represents the dimensionless relation between the inertial forces of the bridge deck and the forces exerted by the fluid. Introducing all variables in $\mathbf{A}$, a complex eigenvalue analysis of the system matrix for increasing wind velocity is performed. The critical velocity is found when one eigenvalue of $\mathbf{A}$ has a positive real part.

4.3. Numerical example

Wilde [95] proposes to find the critical velocity of a bridge with a 2000m span whose properties are stated in Table 4-1.

Notation	Variable	Input
ro_a	Air density in kgf.s^2/m^4	0.125
Bd	Deck width in m	0.2927
m	Mass in kgf.s^2/m^2	0.191
I_a	Mass moment of inertia in kgf.s^2	0.0019345
delta_h	Logarithmic decrement of h mode	0.007
delta_a	Logarithmic decrement of α mode	0.006
omega_h	Fundamental frequency of h mode in rad/s	7.88
omega_a	Fundamental frequency of α mode in rad/s	25.06
a0	A0 (1,1) A0 (1,2) A0 (2,1) A0 (2,2)	1.30E+00 3.53E+00 3.35E-01 8.74E-01
a1	A1 (1,1) A1 (1,2) A1 (2,1) A1 (2,2)	3.38E+00 2.36E+00 7.99E-01 -1.88E-01
d	D (1,1) D (1,2) D (2,1) D (2,2)	3.47E+00 0. 3266975E+01 0. 8526074E+00 0. 8640608E+00
e	E (1,1) E (1,2) E (2,1) E (2,2)	-1.45E-02 7.82E-02 -2.30E-01 2.60E-01
lamb	lamb1 lamb2	0.1911883E+00 0.7477236E+00

Table 4-1 - Data for a 2-DOFs 2000m bridge

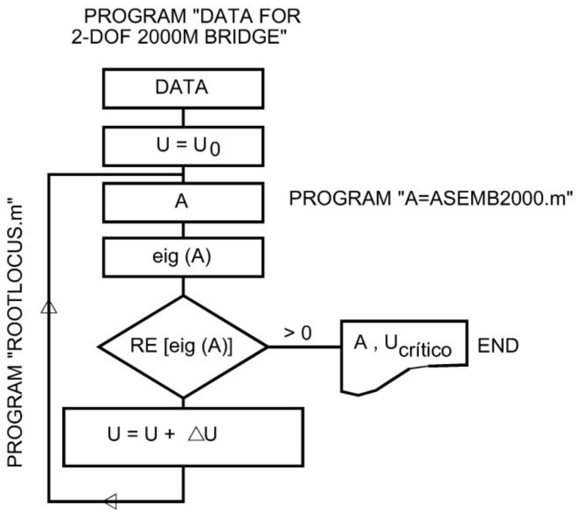

Figure 4-1- Fluxogram 1

4.4. Method to determine the critical velocity of a bridge

In order to calculate the critical velocity of the 2000m bridge, a complex eigenvalue analysis is performed as shown by the fluxogram in Figure 4-1.

The sequence is:

Define the characteristics of the bridge, as in Table 4-1.

Start with the velocity U = U0.

Assemble the state matrix A .

Calculate the eigenvalues of A .

If one eigenvalue of A has a positive real part, print A and U = Ucritical.

If no eigenvalue of A has a positive real part, increase U by ΔU and enter the loop again.

Results of the program “Main_Program_GIBRALTAR.m” written for Wilde’s example are show in Table 4-2. Plots of heaving and pitching frequencies versus wind velocity are presented in Figure 4-2, where all real frequencies were suppressed as being meaningless.

For a small positive real eigenvalue (5.84e-004) the damping ratio approaches zero (-3.29e-005). The critical frequency is 17.8 rad/s. The ratio of amplitudes is the ratio of eigenvectors $|0.010691 - 0.027685i| / |1.5697e{-}006 - 0.047727i| =$ 0.62182. The phase angle is 21.113^0.

Notation	Variable	Result					
U	Critical Velocity	10.21 m/s					
A	State Matrix for 10.21 m/s	-3.3273	-2.3057	-106.5900	-120.5500	-118.3600	-111.4600
		6.6090	-1.5991	96.8000	-375.8200	246.0600	249.3700
		1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
		0.0000	0.0000	-0.5047	2.7272	-6.6697	0.0000
		0.0000	0.0000	-8.0391	9.0545	0.0000	-26.0850
K	Reduced frequency	0.293 x 17.8 rd/s / 10.21 m/s = 0.51					
Results of the complex eigenvalue analysis		Eigenvalue		Damping		Frequency (rad/s)	
		5.84e-004+1.78e+001i		-3.29E-05		1.78E+01	
		5.84e-004-1.78e+001		-3.29E-05		1.78E+01	
		-2.24E+00		1.00E+00		2.24E+00	
		-7.11e+000+9.54e+00i		5.98E-01		1.19E+01	
		-7.11e+000-9.54e+00i		5.98E-01		1.19E+01	
		-2.12E+00		1.10E+01		2.12E+00	

Table 4-2- Results of the complex eigenvalue analysis.

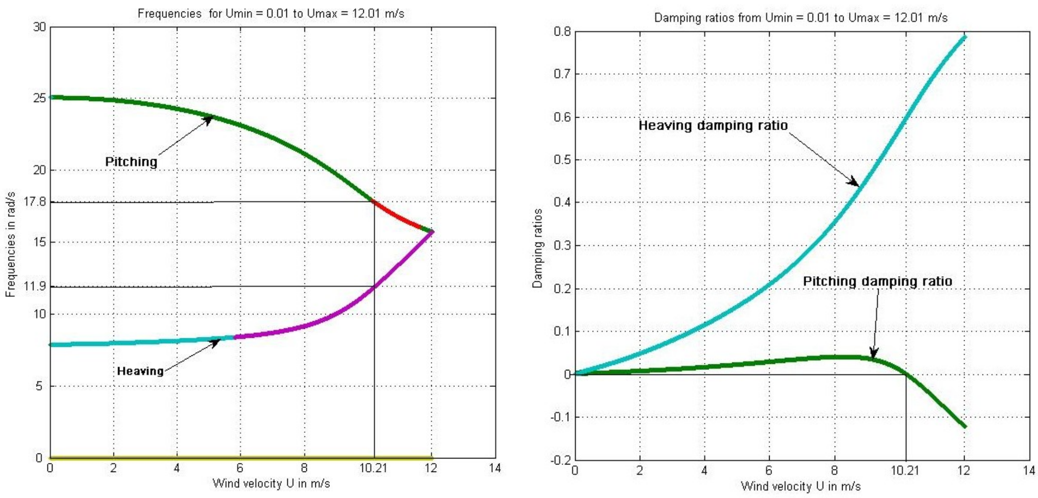

Figure 4-2 - Variation of frequencies and damping ratios versus wind velocity

The frequencies 11.9 and 17.8 rad/s correspond to the critical velocity of 10.21 m/s. The flutter wind velocity corresponds to the point where the damping ratio of pitching changes sign.