Bruno de Barros Mendes Kassar

Three dimensional flow
with suspended spherical particles

DISSERTAÇÃO DE MESTRADO

DEPARTMENTO DE ENGENHARIA MECÂNICA
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Advisor : Prof. Márcio da Silveira Carvalho
Co–Advisor: Prof. Marcos de Oliveira Lage Ferreira

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To my grandmother Lenira de Barros Mendes,
in memoriam.

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Abstract

Kassar, Bruno de Barros Mendes; Carvalho, Márcio da Silveira (Advisor); Ferreira, Marcos de Oliveira Lage (Co-Advisor). **Three dimensional flow with suspended spherical particles**. Rio de Janeiro, 2012. 62p. Dissertação de Mestrado — Departmento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

This work presents a novel implicit and fully coupled formulation for the problem of 3D flows with suspended rigid bodies. This is the main contribution of the work. The formulation was implemented in C++ and tested for the sedimentation problem of one spherical particle. The results indicate plausible physical behavior in spite of being limited by mesh accuracy. The software solves numerically the Navier-Stokes Equations coupled with Rigid Body Dynamics' Equations using the Finite Elements Method. The coupling between fluid and solid domains is done by means of the Fictitious Domain Technique, which avoids mesh generation for every time step. The 3D flow of non particulate flow is also studied in this work and is the basis for the particulate flow formulation.

Keywords

Fluid Mechanics; Finite Elements Method; Navier-Stokes; Fictitious Domain Method.

Resumo

Kassar, Bruno de Barros Mendes; Carvalho, Márcio da Silveira; Ferreira, Marcos de Oliveira Lage. **Escoamento Tridimensional com Partículas Esféricas Suspensas**. Rio de Janeiro, 2012. 62p. Dissertação de Mestrado — Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

Este trabalho apresenta uma nova formulação implícita e totalmente acoplada para o problema de escoamentos tridimensionais com corpos rígidos suspensos. Esta é a principal contribuição deste trabalho. A formulação foi implementada em C++ e testada para o problema de sedimentação de uma partícula esférica. Os resultados indicam comportamento físico plausível apesar de serem limitados por inacurácia de malha. O programa resolve numericamente as Equações de Navier-Stokes acopladas com as Equações da Dinâmica de Corpo Rígido usando o Método de Elementos Finitos. O acoplamento entre os domínios fluido e sólido é feito pela Técnica do Domínio Fictício, que evita a geração de malha a cada passo de tempo. O escoamento tridimensional sem partículas também é estudado neste trabalho e é a base para a formulação do escoamento com partículas.

Palavras–chave

Mecânica dos Fluidos; Método dos Elementos Finitos; Navier-Stokes; Método do Domínio Fictício.

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Nomenclature

$\boldsymbol{\sigma}$	incompressible Newtonian fluid stress tensor
$\boldsymbol{I}$	identity matrix
μ	fluid's dynamic viscosity
ρ_f	fluid's specific mass
ρ_P	specific mass of the particle
$\star$	any direction x, y or z.
$\overrightarrow{\lambda}$	the Lagrange Multipliers vector with components $[\lambda_x, \lambda_y, \lambda_z]^T$
$\overrightarrow{\omega_{P_k}}$	Angular velocity $[\omega_{xP_k}, \omega_{yP_k}, \omega_{zP_k}]^T$ of the particle k
$\overrightarrow{g}$	gravity acceleration vector $[g_x, g_y, g_z]^T$
$\overrightarrow{n_{P_k}}$	normal vector on the particle surface pointing outward the particle
$\overrightarrow{V_{P_k}}$	Linear velocity $[u_{P_k}, v_{P_k}, w_{P_k}]^T$ of the particle k
$\overrightarrow{V}$	velocity vector $[u, v, w]^T$
$\overrightarrow{x}$	the position vector with components $[x, y, z]^T$
L	the the characteristic length of the cavity
N_{DOF}	the total number of degrees of freedom.
N_{elems}	number of elements.
N^x_{elems}	number of elements in x direction.
N^y_{elems}	number of elements in y direction.
N^z_{elems}	number of elements in z direction.
N_{nodes}	number of nodes.
N^x_{nodes}	number of nodes in x direction.
N^y_{nodes}	number of nodes in y direction.
N^z_{nodes}	number of nodes in z direction.
N_{parts}	number of particles.
p	pressure
R_P	radius of the particle
u_l	the lid's velocity on the lid driven cavity problem

Being entirely honest with oneself is a good exercise.

Sigmund Freud, *Letter to Wilhelm Fliess.*