2 Preliminaries

We introduce in this section the basic definitions and the terminology that we use throughout this thesis. In what follows, M is a Riemannian compact manifold without boundary and, for $r \geq 1$, $\mathrm{Diff}^r(M)$ is the space of all C^r diffeomorphisms from M to itself endowed with the usual topology.

The orbit, the forward orbit, and the backward orbit of a set $X \subset M$ with respect to a diffeomorphism f are denoted, respectively, by $\mathcal{O}_f(X)$, $\mathcal{O}_f^+(X)$, and $\mathcal{O}_f^-(X)$.

For every $f \in \mathrm{Diff}^1(M)$ and every open subset U of M, we define the maximal f-invariant set of f in U by

$$\Lambda_f(U) := \bigcap_{n\in\mathbb{Z}} f^n(U).$$

Remark 2.1 In principle, the set $\Lambda_f(U)$ may be empty. However, if there is a point $x \in M$ such that $\mathcal{O}_f(x) \subset U$, then $\mathcal{O}_f(x) \subset \Lambda_f(U)$.

With respect to a diffeomorphism $f \in \mathrm{Diff}^1(M)$, a compact invariant set $\Lambda \subset M$ is said to be:

- *Isolated* or *locally maximal*: If there is an open neighborhood U of Λ such that $\Lambda = \Lambda_f(U)$. Equivalently, Λ is the maximal invariant subset of f in U. Any open neighborhood U of Λ satisfying $\Lambda = \Lambda_f(U)$ is called an *isolating block* of Λ.

- An *attractor*: If there is an open neighborhood U of Λ such that $f(\overline{U}) \subset U$ and $\Lambda = \bigcap_{n\in\mathbb{N}} f^n(U)$. We call Λ a *proper* attractor if $\Lambda \neq M$, and thus $U \neq M$.

- *Transitive*: If there is $x \in \Lambda$ such that its forward orbit $\mathcal{O}_f^+(x)$ is dense in Λ. In our setting, this is equivalent to the following property: Given any pair V_1, V_2 of (relative) nonempty open sets of Λ, there is $n \in \mathbb{Z}$ such that $f^n(V_1) \cap V_2 \neq \emptyset$.

- *Topologically mixing*: If for any pair V_1, V_2 of (relative) open sets of Λ, there is $n \in \mathbb{N}$ such that $f^m(V_1) \cap V_2 \neq \emptyset$ for all $m \geq n$.
- *Robustly transitive set (resp. attractor)*: If there are an isolating block U of Λ and a neighborhood $\mathcal{U}$ of f such that, for every $g \in \mathcal{U}$, the set $\Lambda_g(U)$ is a compact transitive set (resp. attractor) with respect to g.
- *Generically transitive set (resp. attractor)*: If there are an isolating block U of Λ, a neighborhood $\mathcal{U}$ of f, and a residual subset $\mathcal{R} \subset \mathcal{U}$ such that, for every $g \in \mathcal{R}$, the set $\Lambda_g(U)$ is a compact transitive set (resp. attractor) with respect to g.

Remark 2.2 Isolated sets vary, *a priori*, just upper semicontinuously (see Scholium 4.19 for a proof). By an abuse of terminology, we call the set $\Lambda_g(U)$ the *continuation* of the set $\Lambda_f(U)$ when g varies in a small neighborhood of f.

Remark 2.3 An attractor Λ of a diffeomorphism f is an isolated set, so we also denote it by $\Lambda_f(U)$ for some isolating block U of Λ. Observe that if g is close enough to f then the continuation $\Lambda_g(U)$ of $\Lambda_f(U)$ is also an attractor for g. Clearly, if $\Lambda_f(U)$ is proper so is the continuations $\Lambda_g(U)$.

Remark 2.4 In the definition of attractors, some authors requires the additional property that $\Lambda_f(U)$ is transitive. We do not follow this convention. The reason is that we want to talk about the continuation of the attractor as in Remark 2.3, and transitivity is not a robust property.